Dr. Ahmed Soil Mechanics Notes Chapter Four (Stresses in Soil)

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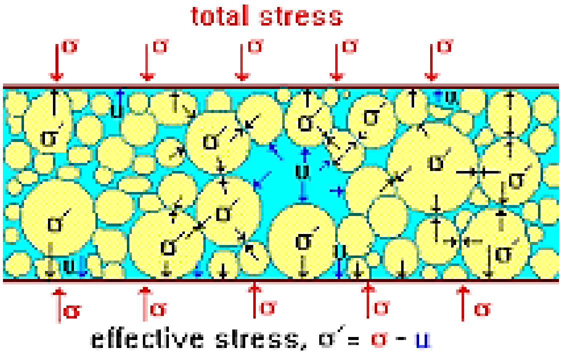
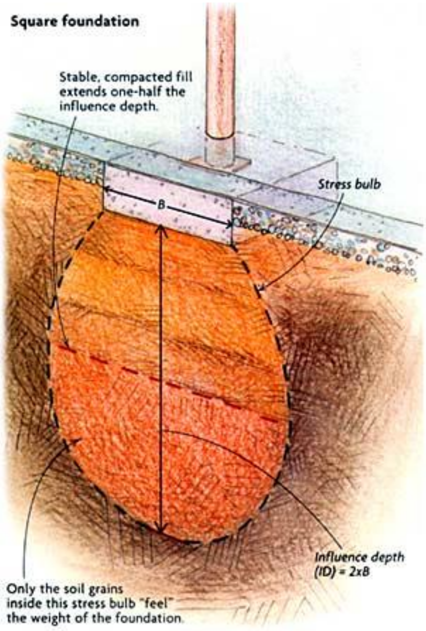
1 author:

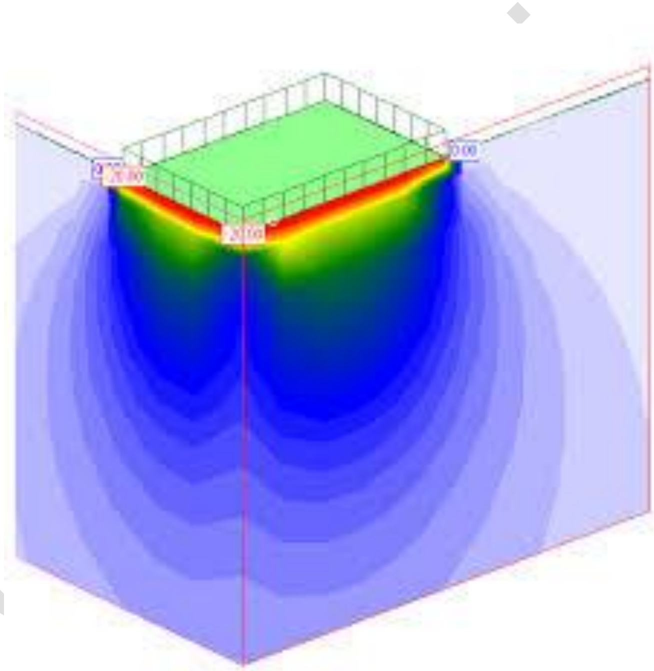
Ahmed Al-Obaidi
Tikrit University



CHAPTER FOUR

STRESSES IN SOIL





Lecture Notes

Soil Mechanics

3rd Class

Assistant Prof.

Dr. Ahmed Al-Obaidi

CHAPTER FOUR
STRESSES IN SOIL

4.1 Introduction

The soils are multiphase systems. In a given volume of soil, the solid particles are distributed randomly with void spaces between. The void spaces are continuous and are occupied by water and/or air. To analyze problems (such as compressibility of soils, bearing capacity of foundations, the stability of embankments, and lateral pressure on earth-retaining structures), It needs to know the nature of the distribution of stress in a given cross section of the soil profile.

In an original soil, it obviously is impossible to keep track of forces at each contact point. Also, it is necessary to use the concept of stress. Stresses within the soil are:

1- **Geostatic stress**: Sub Surface Stresses caused by mass of soil

   A. Vertical stress

   B. Horizontal Stress

2- **Stresses due to surface loading:**

   A. infinitely loaded area (filling)

   B. Point load (concentrated load)

   C. Circular loaded area.

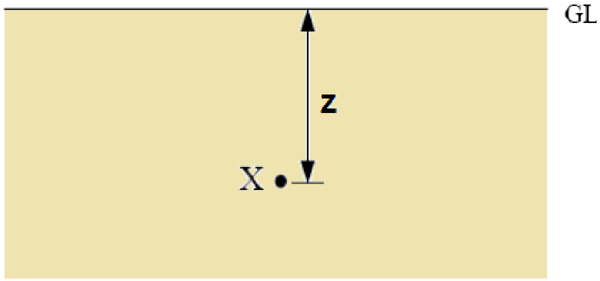
   D. Rectangular loaded area.

4.2 Geostatic Stresses

When the ground surface is horizontal, and when the nature of the soil varies but little in the horizontal direction. In such a situation, the stresses are called *Geostatic Stresses*

4.2.1 Vertical Geostatic Stresses

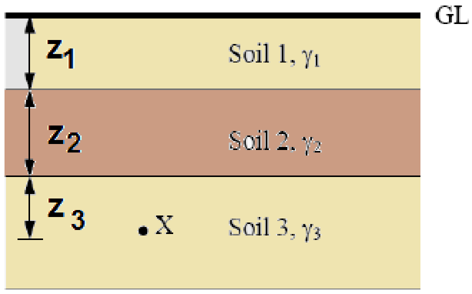
The vertical geostatic stresses at any depth can be computed by considering the weight of soil above the depth. If the unit weight of the soil is constant with depth:



$$\sigma_v = \gamma z$$

where z is the depth of the point considered and γ is the soil unit weight

For layered soil:



$$\sigma_v = \gamma_1 z_1 + \gamma_2 z_2 + \gamma_3 z_3$$

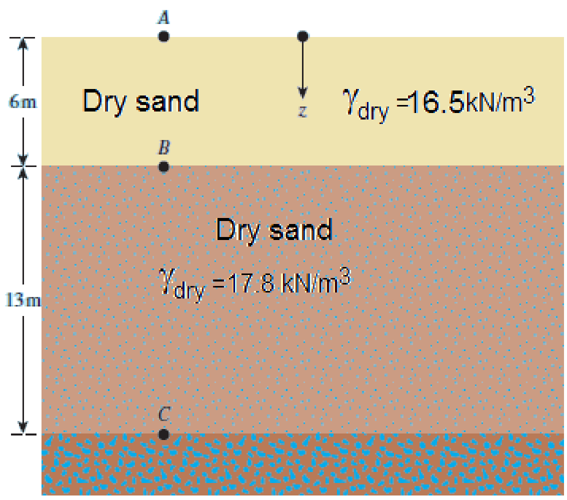
$$\sigma_v = \sum \gamma_i z_i$$

If the unit weight is varied with depth

$$\sigma_v = \int_0^z \gamma_i dz$$

Example (4.1)

For the soil profile, calculate the vertical stresses at points (A), (B), and (C).



Solution

For point (A), z = 0, thus $\sigma_v = 0$

For point (B)

$\sigma_v = \gamma z$ = 16.5 * 6 = 99 kN/m^2

For point (C)

$\sigma_v = \sum \gamma_i z_i$ =16.5*6 +17.8*13 = 330.4 kN/m^2

4.2.2 Effective Vertical Stresses

In saturated soils, the normal stress (σ_v) at any point within the soil mass is shared by the soil grains and the water held within the pores. The component of the normal stress acting on the soil grains, is called effective stress or intergranular stress, and is generally denoted by σ'. The remainder, the normal stress acting on the pore water, is known as pore water pressure or neutral stress and is denoted by (u). Thus, the total stress at any point within the soil mass can be written as:

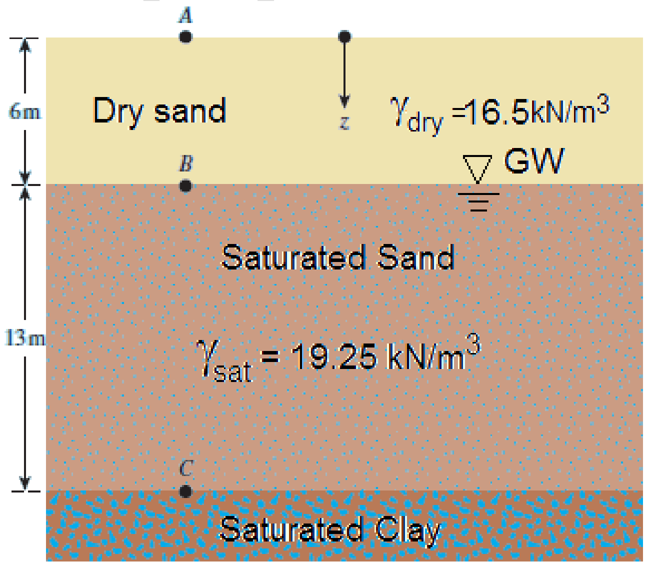
$\sigma = \sigma' + u$

This applies to normal stresses in all directions at any point within the soil mass. In dry soil, there is no pore water pressure and the total stress is the same as effective stress.

In geostatic stresses there is no shear stress in soil, also water cannot carry any shear stress.

Example (4.2)

For the soil profile calculate the vertical total, effective stresses and pore water pressure at points (A), (B), and (C).



Solution

At Point A:

Total stress: $\sigma_{VA} = 0$

Pore water pressure $u_A = 0$

Effective stresses = $\sigma'_{VA} = 0$

At Point B

$\sigma_{vB} = \gamma z$ = 16.5 * 6 = 99 kN/m^2

$u_B = 0$

σ'_{VB} = 99 – 0 = 99 kN/m^2

At Point C

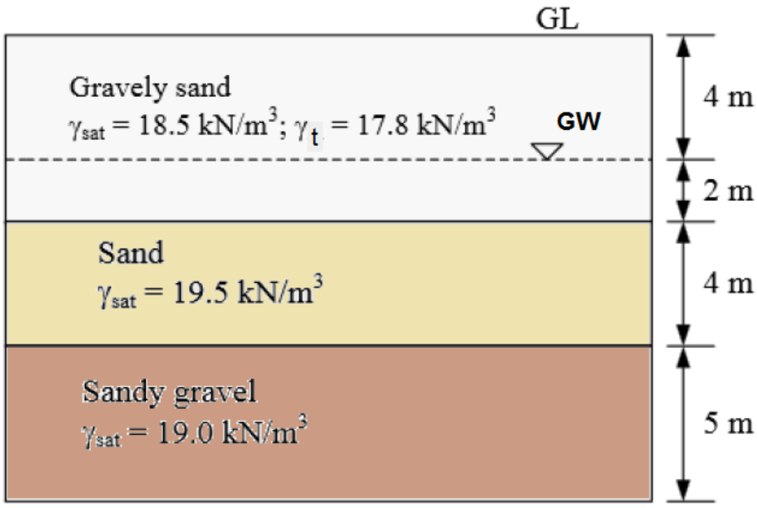
$\sigma_{vC} = \sum \gamma_i z_i$ =16.5*6 +19.25*13 = 349.25 kN/m^2

u_C = 13 * 9.81 = 127.53 kN/m^2

σ'_{VC} = 349.25 – 127.53 = 221.72 kN/m^2

Example (4.3)

Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below:



Solution

Within a soil layer, the unit weight is constant, and therefore the stresses vary linearly. Therefore, it is adequate if we compute the values at the layer interfaces and water table location, and join them by straight lines. At the ground level,

At depth = 0

$\sigma_v = 0$; $\acute{\sigma}_v = 0$; and u=0

At 4 m depth,

$\sigma_v = (4)(17.8) = 71.2$ kPa; u = 0

$\therefore \acute{\sigma}_v = 71.2$ kPa

At 6 m depth, $\sigma_v = (4)(17.8) + (2)(18.5) = 108.2$ kPa

u = (2)(9.81) = 19.6 kPa

$\therefore \acute{\sigma}_v = 108.2 - 19.6 = 88.6$ kPa

At 10 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2$ kPa

u = (6)(9.81) = 58.9 kPa

$\therefore \acute{\sigma}_v = 186.2 - 58.9 = 127.3$ kPa

At 15 m depth,

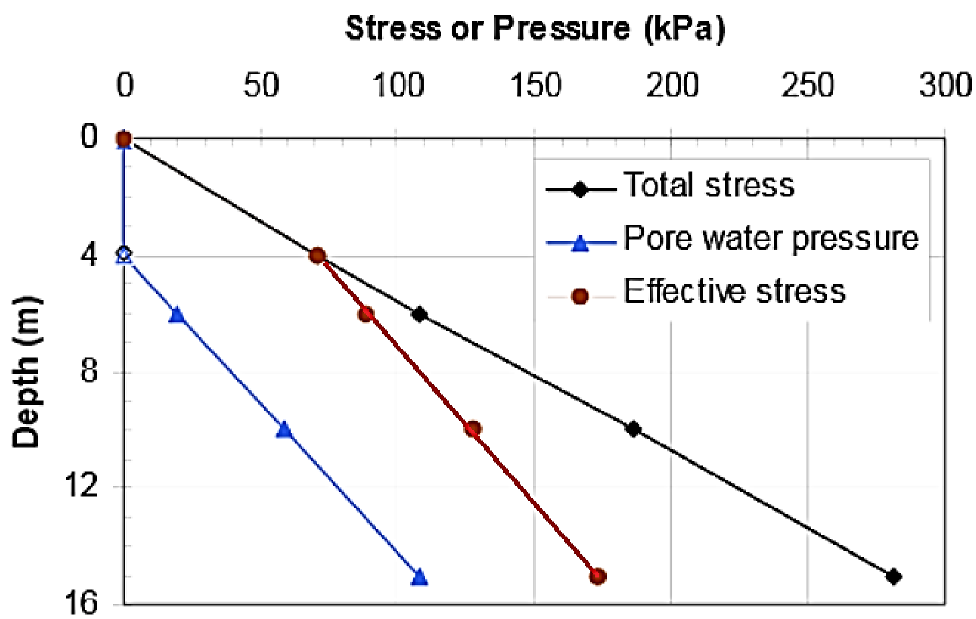
$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2$ kPa

u = (11)(9.81) = 107.9 kPa

$\therefore \acute{\sigma}_v = 281.2 - 107.9 = 173.3$ kPa

The values of σ, u, and σ' computed above are summarized in Table

Depth (m)	σ_v (kPa)	U (kPa)	σ'_v (kPa)
0	0	0	0
4	71.2	0	71.2
6	108.2	19.6	88.6
10	186.2	58.9	127.3
15	281.2	107.9	173.3



Variation of σ_v, u and $\acute{\sigma}_v$ with depth

<u>Example (4.4)</u>

Refer to Example 4.2. How high should the water table rise so that the effective stress at C is 190 kN/m^2? Assume γ_{sat} to be the same for both layers (i.e., 19.25 kN/m^3).

<u>Solution:</u> Let the groundwater table rise be (h) above the present groundwater table

$$\sigma_C = (6-h)\gamma_{dry} + h\gamma_{sat} + 13\gamma_{sat}$$
$$u = (h+13)\gamma_w$$

So

$$\sigma'_C = \sigma_c - u = (6-h)\gamma_{dry} + h\gamma_{sat} + 13\gamma_{sat} - h\gamma_w - 13\gamma_w$$
$$= (6-h)\gamma_{dry} + h(\gamma_{sat} - \gamma_w) + 13(\gamma_{sat} - \gamma_w)$$

$$190 = (6-h)16.5 + h(19.25 - 9.81) + 13(19.25 - 9.81)$$
$$h = 4.49 \text{ m}$$

4.2.3 Effective Horizontal Stresses

The horizontal geostatic stress can be computed as following:

$\sigma'_h = k_o \sigma'_v$

k_o is the coefficient of lateral stress

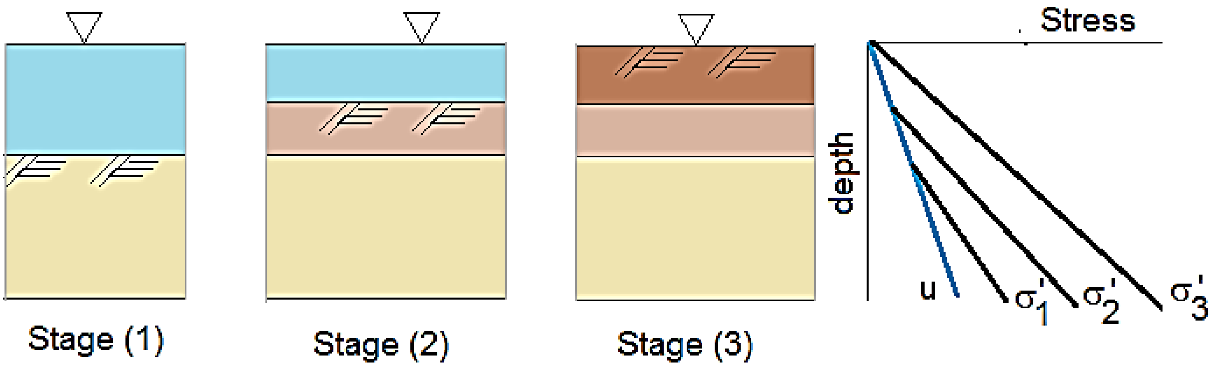
In sand soil and normally consolidated clay $k_o < 1.0 = 1 - \sin\phi$, where ϕ is the angle of internal friction of soil

In over consolidated clay $k > 1.0$

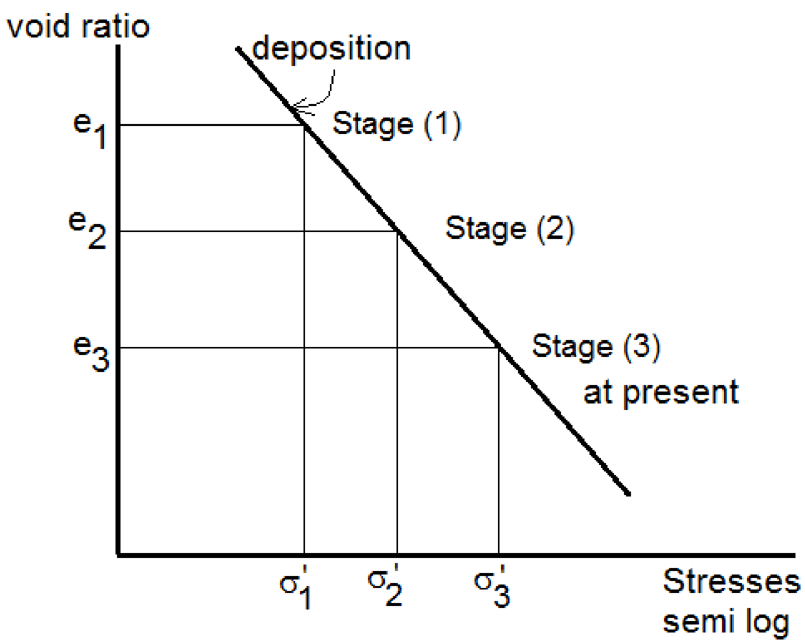
The stress to which soil has been subjected during its formation to the present time referred as stress history. During deposition, effective stress increased as more soil particle are placed, and during effective erosion, stress decreased as the soil particles are removed. Due to this, there are two types of soils:

Normally consolidated clay and sand:

This soil has undergone deposition only if the water table is assumed at the ground level, the vertical and horizontal effective stresses are increased, and the void ratio of the soil reduced the plot of void ratio versus effective stress

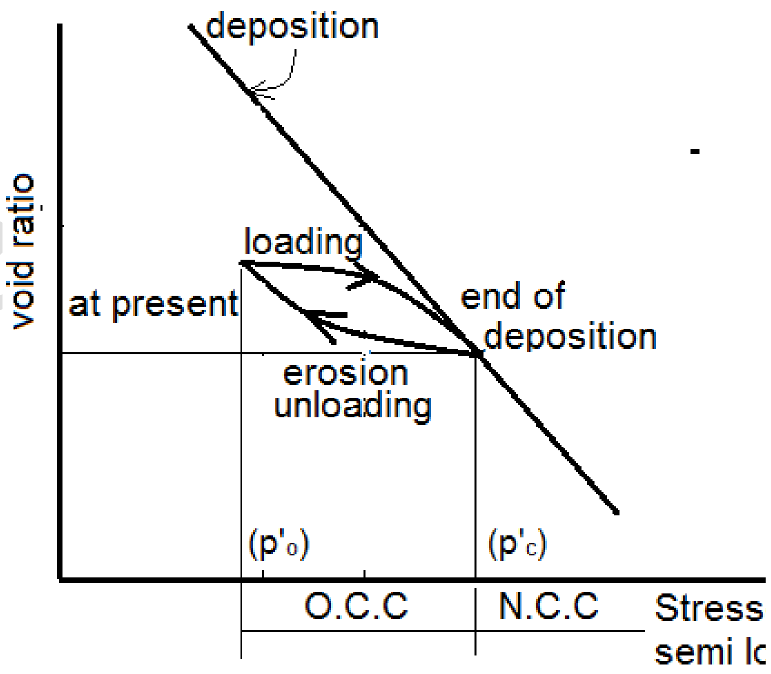


on the semi-log scale is a usually a straight line. During deposition the mineral grains of the soil elements will be rearranged and became closer, the effective stress at the stage will be maximum (p'$_o$).



Overconsolidated Clay:

In this case, the soil has been subjected to effective stress in its past stress history. (p'_c) is larger than the effective stress at present effective stress (p'_o)



When the soil is reloaded from (p'_o), it reached (p'_c).

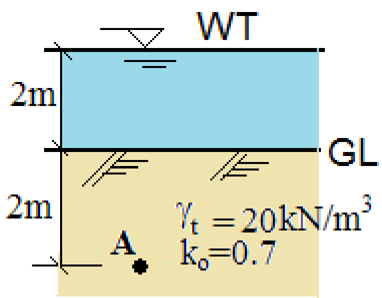
At this stage the soil is over-consolidated.

The over-consolidated ratio (O.C.R) = p'_c / p'_o

- Normally consolidated clay (O.C.R) = 1.0
- Lightly over-consolidated clay (O.C.R) = 1.5 - 3
- over-consolidated clay (O.C.R) = > 4

Example (4.5)

Compute the vertical and horizontal total and effective stresses and pore water pressure at element (A)



Solution

In this example, the water above soil is an additional load on the soil thus:

Total vertical stress at A, σ_v = 9.81* 2 + 20*2 = 59.62 kN/m^2

Pore water pressure at A u = 9.81* 4 = 39.24 kN/m^2

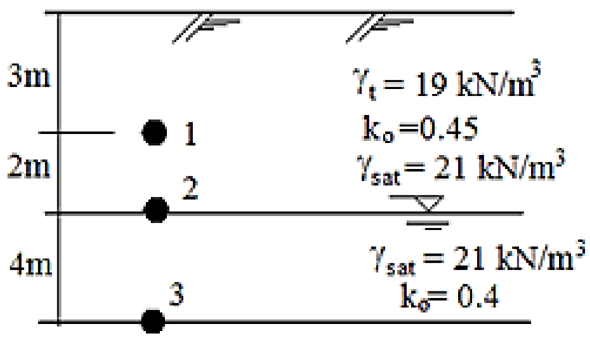
Effective vertical stress at A σ'_v = 59.62 - 39.24 = 20.38 kN/m^2

Effective horizontal stress at A, $\sigma'_h = K\sigma'_v$ = 0.7 * 20.8 = 14.266 kN/m^2

Total horizontal stress at A, $\sigma_h = \sigma'_h + u$ = 14.266 + 39.24 = 53.51 kN/m^2

Example (4.6)

Compute the vertical and horizontal total and effective stresses and pore water pressure at element (1), (2), and (3).



Solution

Point (1)

Total vertical stress at point (1), σ_v = 19* 3 = 57 kN/m^2

Pore water pressure at (1), u = 9.81* 0 = 0 kN/m^2

Effective vertical stress at (1), σ'_v = 57 - 0 = 57 kN/m^2

Effective horizontal stress at (1), $\sigma'_h = k_o\sigma'_v$ = 0.45 * 57 = 25.65 kN/m^2

Total horizontal stress at (1), $\sigma_h = \sigma'_h + u$ = 25.65 + 0 = 25.65 kN/m^2

Point (2)

Total vertical stress at point (2), σ_v = 19* (3+2) = 95 kN/m^2

Pore water pressure at (2), u = 9.81* 0 = 0 kN/m^2

Effective vertical stress at (2), σ'_v = 95 - 0 = 95 kN/m^2

Effective horizontal stress at (2), $\sigma'_h = k_o \sigma'_v$ = 0.45 * 95 = 42.72 kN/m^2

Total horizontal stress at (2), $\sigma_h = \sigma'_h$ + u = 42.72 + 0 = 42.72 kN/m^2

Point (3)

Total vertical stress at point (3), σ_v = 95 +21*4 = 179 kN/m^2

Pore water pressure at (3), u = 9.81* 4 = 39.24 kN/m^2

Effective vertical stress at (3), σ'_v = 179 – 39.24 = 139.76 kN/m^2

Effective horizontal stress at (3), $\sigma'_h = k_o \sigma'_v$ = 0.40 * 139.76 = 55.9 kN/m^2

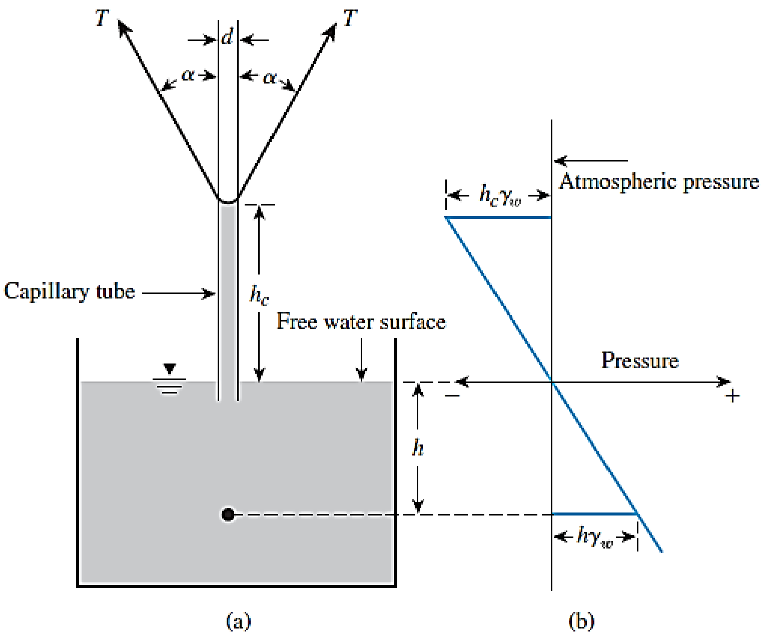
Total horizontal stress at (3), $\sigma_h = \sigma'_h$ + u= 55.9 + 39.24 = 95.14 kN/m^2

4.3 Capillary Rise in Soils

The continuous void spaces in soil can behave as bundles of capillary tubes of variable cross section. Because of surface tension force, water may rise above the phreatic surface. Figure shows the fundamental concept of the height of rise in a capillary tube. The height of rise of water in the capillary tube can be given by summing the forces in the vertical direction, or

$$\left(\frac{\pi}{4} d^2\right) h_c \gamma_w = \pi d T \cos \alpha$$

$$h_c = \frac{4T \cos \alpha}{d \gamma_w}$$



where

T=surface tension (force / length)

α = angle of contact

d = diameter of the capillary tube

γ_w = unit weight of water

For pure water and clean glass, α = 0. Thus, above Eq. becomes

$$h_c = \frac{4T}{d \gamma_w}$$

For water, T = 72 m.N/m. the height of capillary rise is: $h_c \propto \frac{1}{d}$

Thus, the smaller the capillary tube diameter, the larger the capillary rise. The concept of the capillary rise can be applied to soils; one must realize that the capillary tubes formed in soils because of the continuity of voids have variable cross-sections. After the lapse of a given amount of time, the variation of the degree of saturation with the height of the soil column caused by capillary rise. The degree of saturation is about 100%. The maximum height of capillary rise is

$$h\ (\text{mm}) = \frac{C}{eD_{10}}$$

where

D_{10} = effective size (mm)

e = void ratio

C = a constant that varies from 10 to 50 mm^2

The table below shows the approximate range of capillary rise that is encountered in various types of soils.

Approximate Range of Capillary Rise in Soils

Soil type	Range of capillary rise m
Coarse sand	0.1–0.2
Fine sand	0.3–1.2
Silt	0.75–7.5
Clay	7.5–23

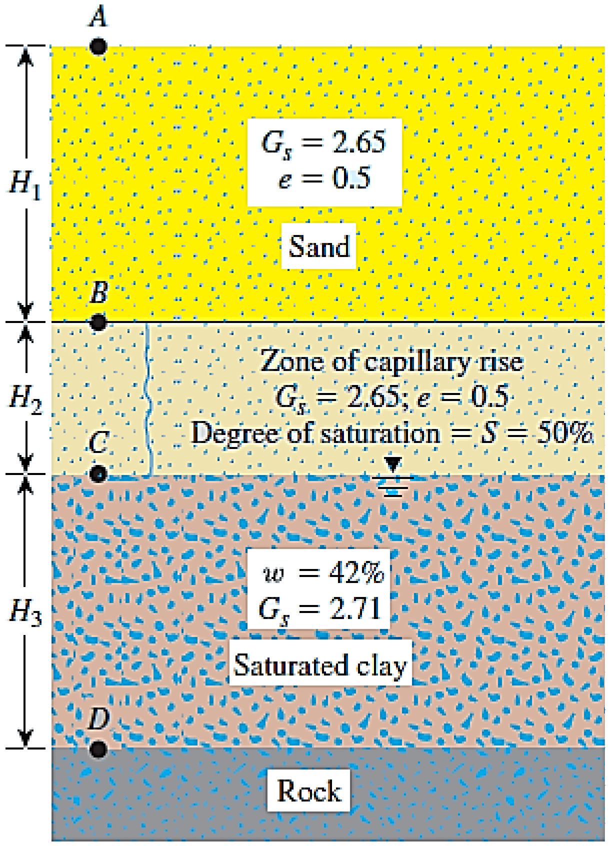
Effective Stress in the Zone of Capillary Rise

The general relationship between total stress, effective stress and pore water pressure was given as $\sigma = \sigma' + u$

The pore water pressure (u) at a point in a layer of soil fully saturated by a capillary rise is equal to $-\gamma_w h$ (h = height of the point under consideration measured from the groundwater table) with the atmospheric pressure taken as a datum.

Example (4.7)

A soil profile is shown in. Given: H_1 = 1.83 m, H_2 = 0.91 m, H_3 = 1.83 m. Plot the variation of σ, σ', and u with depth.



Solution

Determination of Unit Weight

Dry sand:

$$\gamma_{d(\text{sand})} = \frac{G_s\gamma_w}{1+e} = \frac{(2.65)(9.81)}{1+0.5}$$

$$= 17.33 \text{ kN/m}^3$$

Moist sand:

$$\gamma_{\text{sand}} = \frac{(G_s + Se)\gamma_w}{1+e} = \frac{[2.65 + (0.5)(0.5)]9.81}{1+0.5}$$

$$= 18.97 \text{ kN/m}^3$$

Saturated clay:

$$e = \frac{G_s w}{S} = \frac{(2.71)(0.42)}{1.0} = 1.1382$$

$$\gamma_{\text{sat(clay)}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.71 + 1.1382)9.81}{1 + 1.1382} = 17.66 \text{ kN/m}^3$$

Calculation of Stress

At the ground surface (i.e., point A):

$$\sigma = 0$$
$$u = 0$$
$$\sigma' = \sigma - u = 0$$

At depth H_1 (i.e., point B):

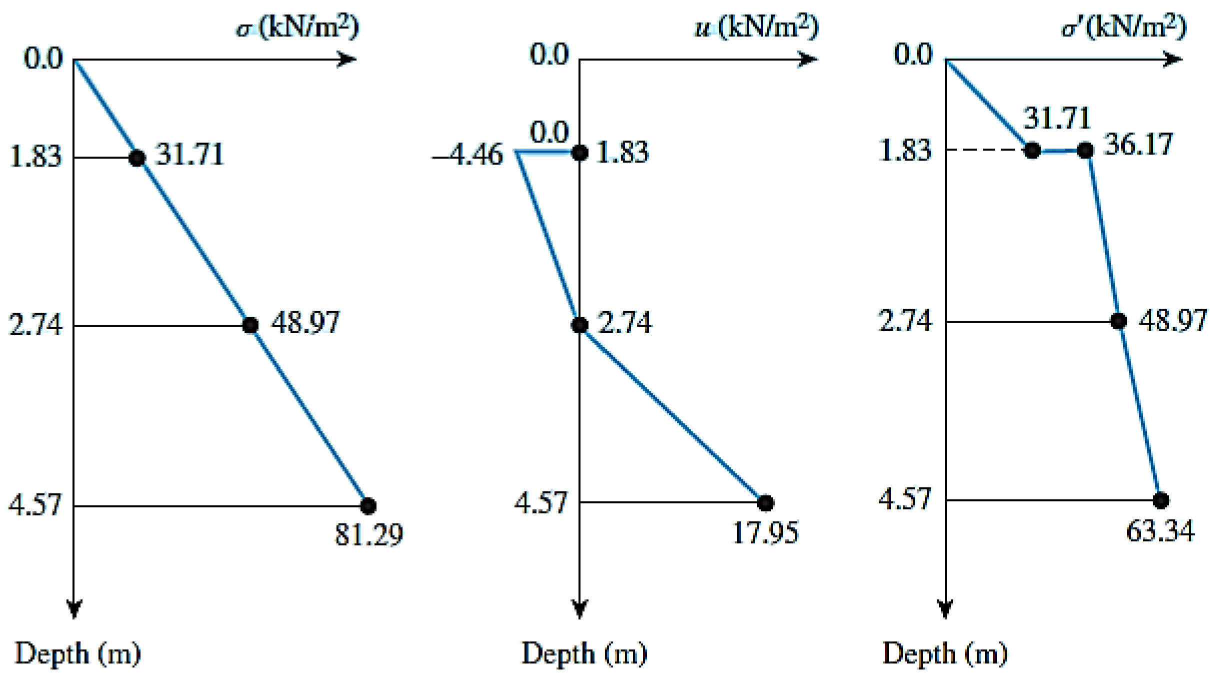
$$\sigma = \gamma_{d(\text{sand})}(1.83) = (17.33)(1.83) = \mathbf{31.71\ kN/m^2}$$
$$u = \mathbf{0} \text{ (immediately above)}$$
$$u = -(S\gamma_w H_2) = -(0.5)(9.81)(0.91) = -\mathbf{4.46\ kN/m^2} \text{ (immediately below)}$$
$$\sigma' = 31.71 - 0 = \mathbf{31.71\ kN/m^2} \text{ (immediately above)}$$
$$\sigma' = 31.71 - (-93.6) = \mathbf{36.17\ kN/m^2} \text{ (immediately below)}$$

At depth $H_1 + H_2$ (i.e., at point C):

$$\sigma = (17.33)(1.83) + (18.97)(0.91) = \mathbf{48.97\ kN/m^2}$$
$$u = \mathbf{0}$$
$$\sigma' = 48.97 - 0 = \mathbf{48.97\ kN/m^2}$$

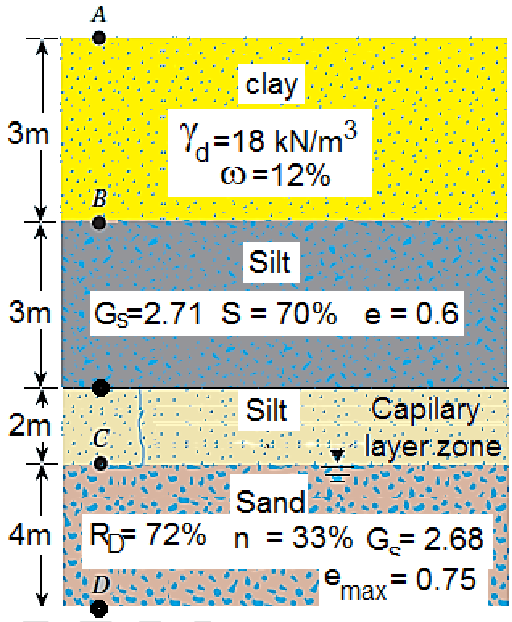
At depth $H_1 + H_2 + H_3$ (i.e., at point D):

$$\sigma = 48.97 + (17.66)(1.83) = \mathbf{81.29\ kN/m^2}$$
$$u = 1.83\gamma_w = (1.83)(9.81) = \mathbf{17.95\ kN/m^2}$$
$$\sigma' = 81.29 - 17.95 = \mathbf{63.34\ kN/m^2}$$



Example (4.8)

For the soil profile shown draw σ, σ', and u with depth



Solution

First find the unit of the soils

Clay layer $\gamma_t = (1+\omega)\,\gamma_d$

$= (1+0.12)*18 = 20.16$ kN/m^3

Silt layer, $\gamma_t = \frac{G_s+S.e}{1+e}\gamma_w$

$$\gamma_t = \frac{2.71+0.7*0.6}{1+0.6}*9.81 = 19.2 \text{ kN/m}^3$$

At capillary zone

$$\gamma_{sat} = \frac{G_s+S.e}{1+e}\gamma_w = \frac{2.71+1*0.6}{1+0.6}*9.81 = 20.3 \text{ kN/m}^3$$

Sand layer

$$e_{min} = \frac{n}{1-n} = \frac{0.33}{1-0.33} = 0.493$$

$$R_D = \frac{e_{max} - e_{field}}{e_{max} - e_{min}} = 0.72 = \frac{0.75 - e_{field}}{0.75 - 0.493} \rightarrow e_{field} = 0.565$$

$$\gamma_{sat} = \frac{G_s+S.e}{1+e}\gamma_w = \frac{2.68+1*0.565}{1+0.565}*9.81 = 20.34 \text{ kN/m}^3$$

Calculation of stresses

At point A

Total vertical stress, σ_v = 20.16 * 3 = 60.48 kN/m^2

Pore water pressure, u = 9.81* 0 = 0 kN/m^2

Effective vertical stress, σ'_v = 60.48 - 0 = 60.48 kN/m^2

At point B

Total vertical stress, σ_v = 20.16 * 3 + 19.2 * 3 = 118.08 kN/m^2

Pore water pressure, u = - 9.81* 2 = - 19.62 kN/m^2

Effective vertical stress, σ'_v = 118.08 + 19.62 = 137.7 kN/m^2

At point C

Total vertical stress, σ_v =20.16 * 3 +19.2 * 3 +20.3 * 2 = 158.68 kN/m^2

Pore water pressure, u = 9.81* 0 = 0 kN/m^2

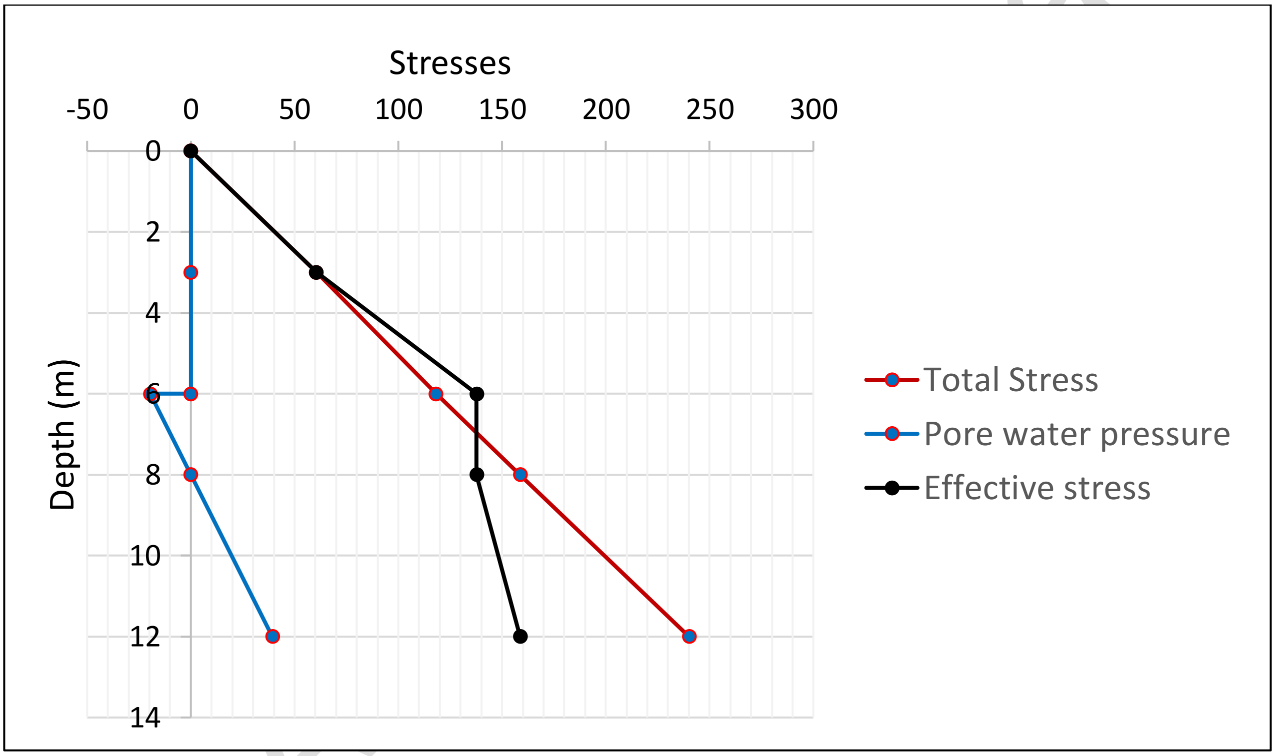
Effective vertical stress, σ'_v = 158.68 - 0 = 158.68 kN/m^2

At point D

Total vertical stress, σ_v =158.68 + 20.35 * 4 = 240.08 kN/m^2

Pore water pressure, u = 9.81* 4 = 39.24 kN/m^2

Effective vertical stress, σ'_v = 240.08 - 39.24 = 200.84 kN/m^2



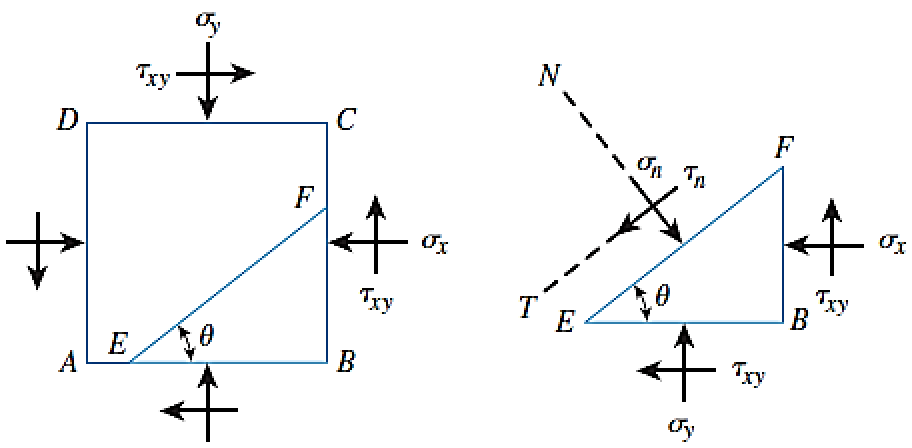
Important Note

The effective stress principle is probably the most important concept in geotechnical engineering. The compressibility and shearing resistance of a soil depend largely on the effective stress. Thus, the concept of effective stress is significant in solving geotechnical engineering problems, such as the lateral earth pressure on retaining structures, the load-bearing capacity and settlement of foundations, and the stability of earth slopes.

4.4 Normal and Shear Stresses on a Plane

This section is a brief review of the basic concepts of normal and shear stresses on a plane that can be found in any course on the mechanics of materials.

The Figure shows a two-dimensional soil element that is being subjected to normal and shear stresses ($\sigma_y > \sigma_x$). To determine the normal stress and the shear stress on a plane EF that makes an angle θ with the plane AB, the free body diagram of EFB shown. Let σ_n and



(a) (b)

τ_n be the normal stress and the shear stress respectively, on the plane EF.

From geometry. $\overline{EB} = \overline{EF}\cos\theta$ and $\overline{FB} = \overline{EF}\sin\theta$

Summing the components of forces that act on the element in the direction of N and T,

$$\sigma_n(\overline{EF}) = \sigma_x(\overline{EF})\sin^2\theta + \sigma_y(\overline{EF})\cos^2\theta + 2\tau_{xy}(\overline{EF})\sin\theta\cos\theta$$

or $\sigma_n = \sigma_x\sin^2\theta + \sigma_y\cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

or

$$\boxed{\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta}$$

Again,

$$\tau_n(\overline{EF}) = -\sigma_x(\overline{EF})\sin\theta\cos\theta + \sigma_y(\overline{EF})\sin\theta\cos\theta - \tau_{xy}(\overline{EF})\cos^2\theta + \tau_{xy}(\overline{EF})\sin^2\theta$$

or $\tau_n = \sigma_y\sin\theta\cos\theta - \sigma_x\sin\theta\cos\theta - \tau_{xy}(\cos^2\theta - \sin^2\theta)$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

If τ_n equal to zero: $$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_y - \sigma_x}$$

For given values of τ_{xy}, σ_y and σ_x will give two values of θ that are 90° apart. This means, that there are two planes that are at right angles to each other on which the shear stress is zero. Such planes are called **Principal Planes**. The normal stresses that act on the principal planes are referred to as **Principal Stresses**.

Principal stresses: the normal stresses acting on principal planes, the largest principal stress is called **Major Principal Stress** (σ_1), and the smallest principal stresses is called **Minor Principal Stress** (σ_3). The third is called the intermediate principal stress (σ_2)

In isotropic soils $\sigma_3 = \sigma_2$

In anisotropic soil $\sigma_3 \neq \sigma_2$

Isotropic soil: soils that have similar properties at a given location at al planes of all directions

Principal Planes: three planes which is normally stresses act on it and *No Shear Stress*

In Geostatic Condition:

k < 1.0, $\sigma_v = \sigma_1$, $\sigma_h = \sigma_3$

k > 1.0, $\sigma_v = \sigma_3$, $\sigma_h = \sigma_1$

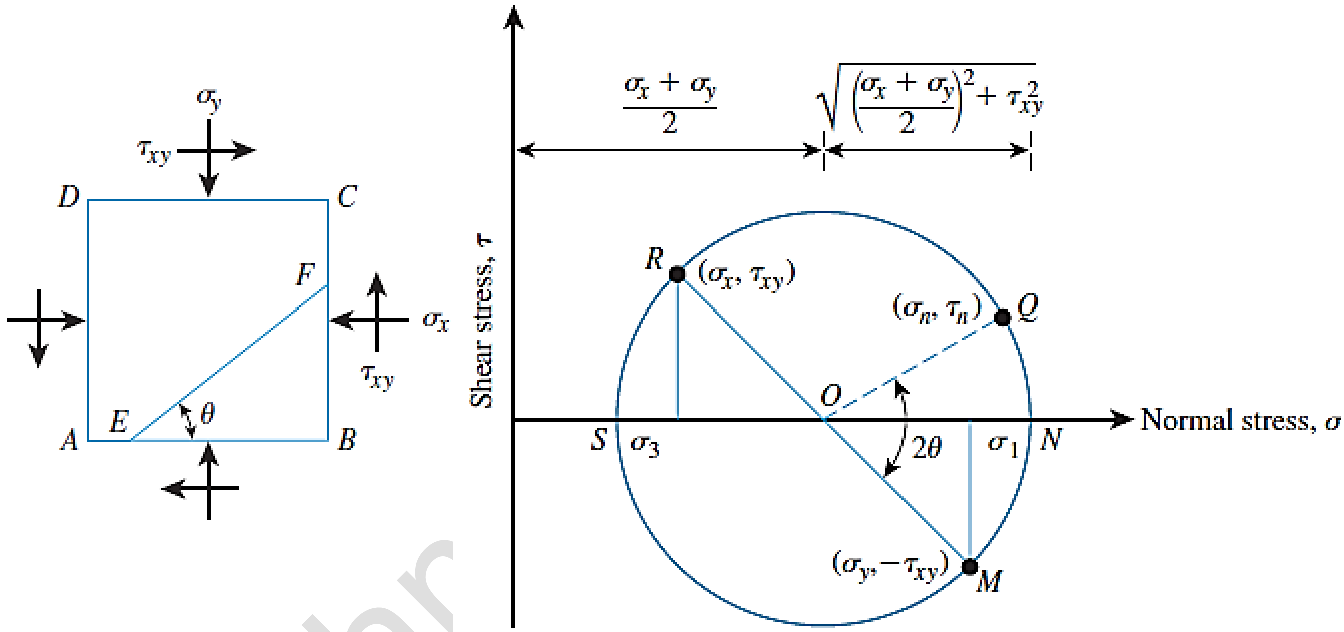
k < 1.0, $\sigma_v = \sigma_h = \sigma_1 = \sigma_3$

The normal stress and shear stress that act on any plane can also be determined by plotting a Mohr's circle, as shown in Figure. The following sign conventions are used in Mohr's circles:

Compressive normal stresses are taken as positive.

Shear stresses are considered positive if they act on opposite faces of the element in such a way that they tend to produce a counterclockwise rotation.

The angle θ is positive when measured counterclockwise from major principal plane.



For plane AD of the soil element shown in Figure, normal stress equals $+\sigma_x$ and shear stress equals $+\tau_{xy}$. For plane AB, normal stress equals $+\sigma_y$ and shear stress equals $-\tau_{xy}$.

The points R and M in Figure represent the stress conditions on planes AD and AB, respectively. O is the point of intersection of the normal stress axis with the line RM. The circle MNQRS drawn with θ as the center and OR as the radius is the Mohr's circle for the stress conditions considered. The radius of the Mohr's circle is equal to:

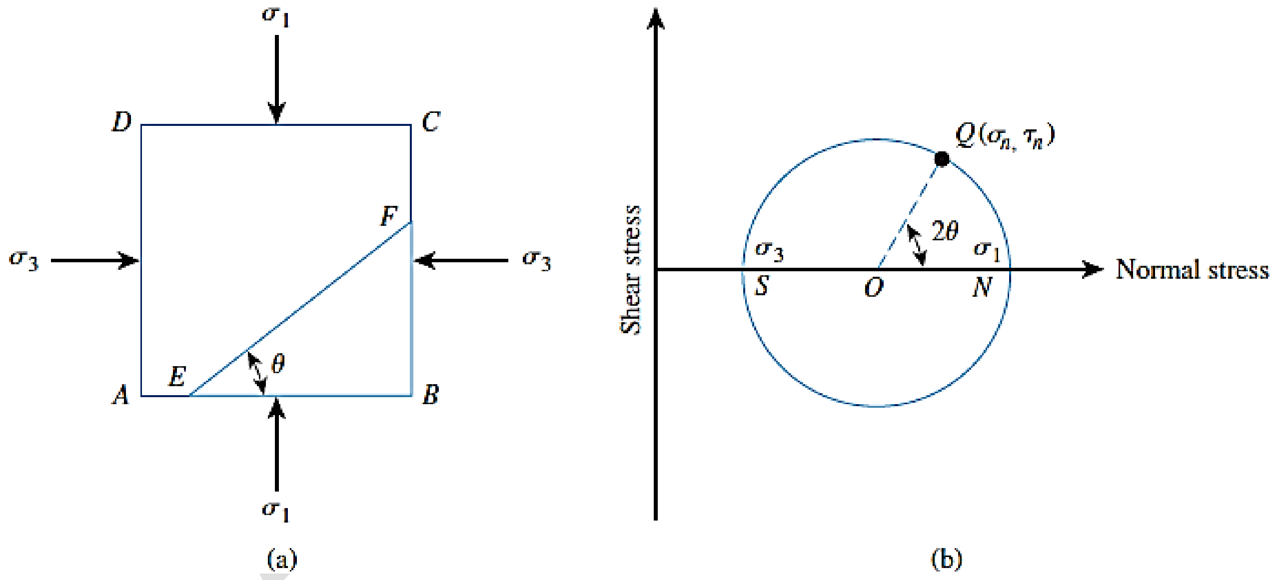
$$\sqrt{\left[\frac{(\sigma_y - \sigma_x)}{2}\right]^2 + \tau_{xy}^2}$$

The stress on plane EF can be determined by moving an angle 2θ (which is twice the angle that the plane EF makes in a counterclockwise direction with plane AB in a counterclockwise direction from point M along the circumference of the Mohr's circle to reach point Q. The abscissa and ordinate of point Q, respectively, give the normal stress an and the shear stress $+\tau_n$ on plane EF.

The abscissa of point N is equal to σ_1, and the abscissa for point S is σ_3.

As a special case, if the planes AB and AD were major and minor principal planes, the normal stress and the shear stress on plane EF could be found by substituting $\tau_{xy} = 0$.

If $\sigma_y = \sigma_1$ and $\sigma_x = \sigma_3$ Thus,



Example (4.9)

The magnitudes of stresses are σ_1 =120 kN/m^2, τ_{xy} =40 kN/m^2, σ_y =300 kN/m^2, and θ = 20°. Determine

a. Magnitudes of the principal stresses.

b. Normal and shear stresses on plane AB.

Solution

(a)

$$\left.\begin{matrix}\sigma_3\\ \sigma_1\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{300 + 120}{2} \pm \sqrt{\left[\frac{300 - 120}{2}\right]^2 + (-40)^2}$$

$$\sigma_1 = \mathbf{308.5\ kN/m^2}$$

$$\sigma_3 = \mathbf{111.5\ kN/m^2}$$

(b)

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\theta + \tau \sin 2\theta$$

$$= \frac{300 + 120}{2} + \frac{300 - 120}{2} \cos (2 \times 20) + (-40) \sin (2 \times 20)$$

$$= \mathbf{253.23\ kN/m^2}$$

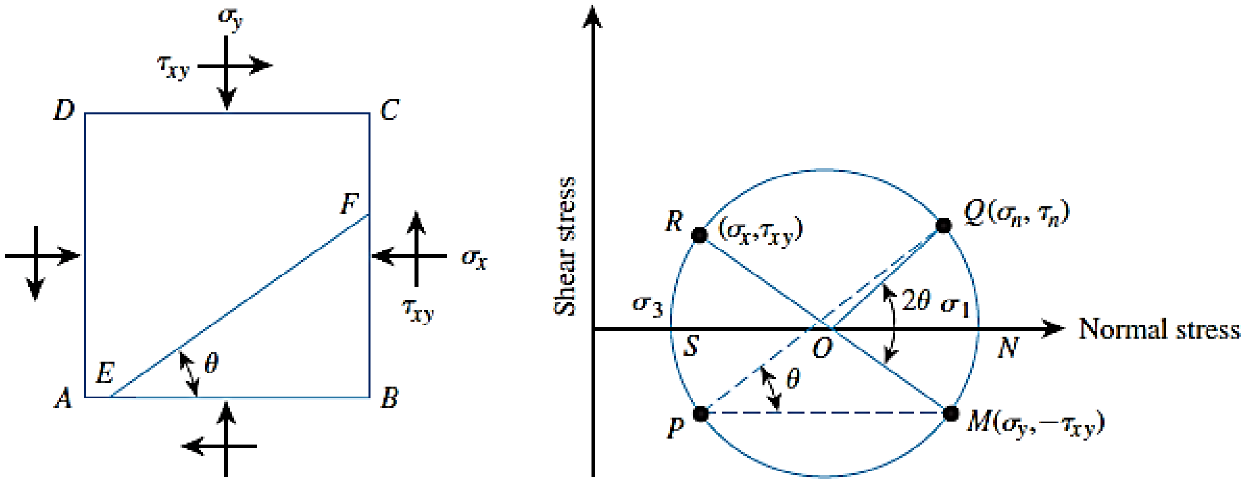
$$\tau_n = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta - \tau \cos 2\theta$$

$$= \frac{300 - 120}{2} \sin (2 \times 20) - (-40) \cos (2 \times 20)$$

$$= \mathbf{88.40\ kN/m^2}$$

The Pole Method of Finding Stresses along a Plane

Another important technique of finding stresses along a plane from a Mohr's circle is the pole method, or the method of origin of planes. This is demonstrated in Figure.

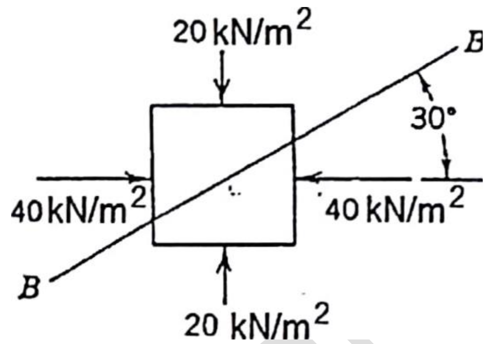


(a) (b)

In this method draw a line from a known point on the Mohr's circle parallel to the plane on which the state of stress acts. The point of intersection of this line with the Mohr's circle is called the pole. This is a unique point for the state of stress under consideration.

For example, the point M on the Mohr's circle in Figure represents the stresses on the plane AB. The line MP is drawn parallel to AB. Therefore, point P is the pole (origin of planes) in this case. To find the stresses on a plane EF, draw a line from the pole parallel to EF. The point of intersection of this line with the Mohr's circle is Q. The coordinates of Q give the stresses on the plane EF. (Note: From geometry, angle QOM is twice the angle QPM.)

Example (4.10)



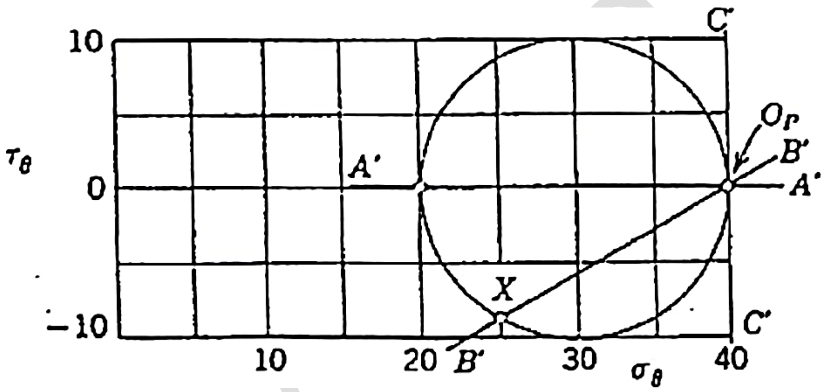
For a given in figure find the stresses at plane B-B

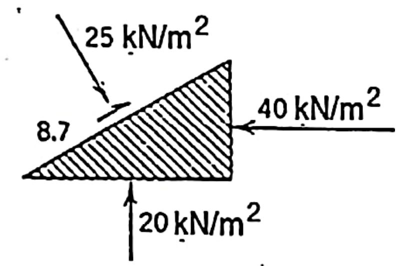
Case one: Given σ_1 and σ_3, **required** σ_θ and τ_θ

Solution

1. locate points with co-ordinates (40,0) and (20,0)
2. Draw circle, using these points to defined diameter, diameter location = (40+20)/ 2 = 30 kN/m^2, thus center location is (30, 0).
3. Draw line A' A' through point (20, 0) and parallel to plane on which stress (20, 0) acts.
4. Intersection of line A' A' with Mohr's circle at point (40,0) is origin of planes
5. Draw line B'B' through point O_P parallel to BB
6. Read coordinates of point X where B'B' intersect Mohr circle

σ_θ = 25 kN/m^2, τ_θ = - 8.7 kN/m^2





Alternate Solution, Step 1 and Step 2 same as above

3. Draw line C'C' through (40, 0) parallel to plane on which stress (40, 0) acts. C'C' is vertical
4. C'C' intersects Mohr circle only at (40,0), so this is Op, Step 5 and 6 same as above

By using equations

$$\sigma_\theta = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

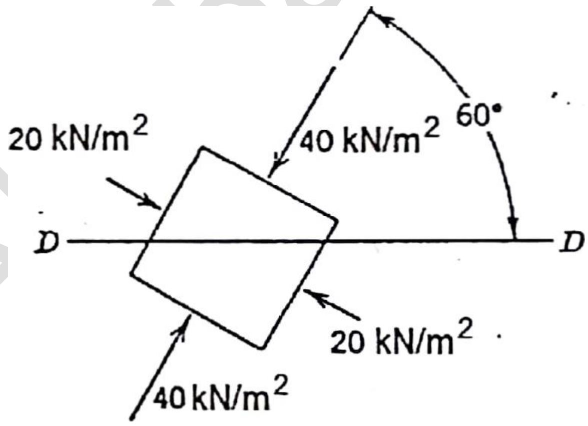
$$\sigma_\theta = \frac{40+20}{2} + \frac{40-20}{2}\cos(2*120) = 30 + (-5) = 25$$

$$\tau = \frac{(\sigma_1 - \sigma_3)}{2}\sin 2\theta$$

$$\tau = \frac{(40 - 20)}{2}\sin(2*120) = -8.66\ kN/m^2$$

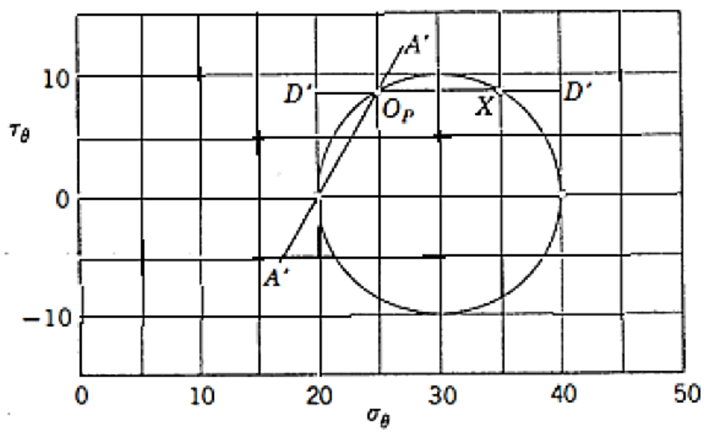
<u>Example (4.11)</u>

For a given in figure find the stresses at plane D-D

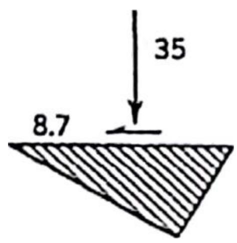


<u>Solution</u>

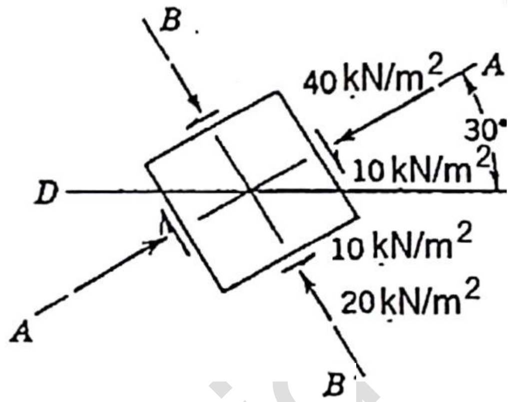
1. locate points with co-ordinates (40,0) and (20,0)
2. Draw Mohr circle, using these points to defined diameter, diameter location = (40+20)/ 2 = 30 kN/m^2, thus center location is (30, 0).
3. Draw line A' A' through point (20, 0) and parallel to plane on which stress (20, 0) acts.
4. Intersection of line A' A' with Mohr's circle gives origin of planes
5. Draw line D'D' through point o O_P parallel to DD
6. Intersection X give stresses



σ_X = 35 kN/m^2, τ_{xy} = 8.7 kN/m^2



Example (4.12)

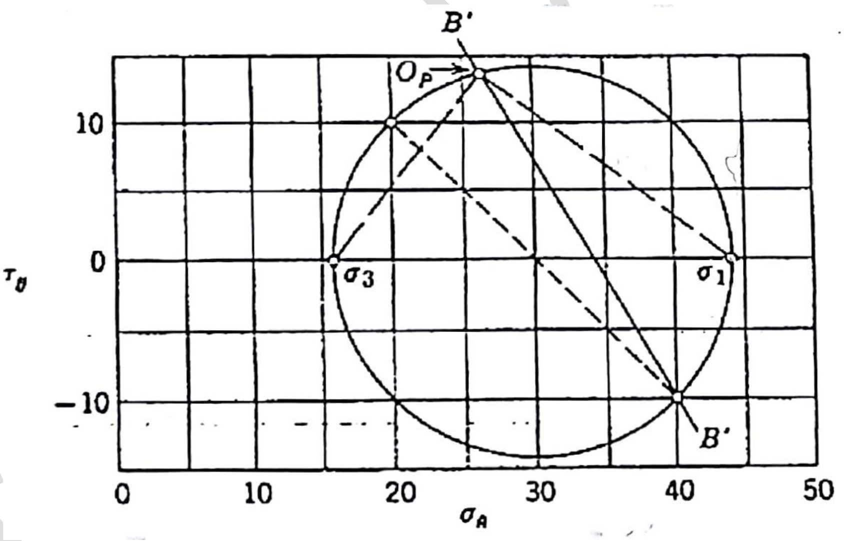


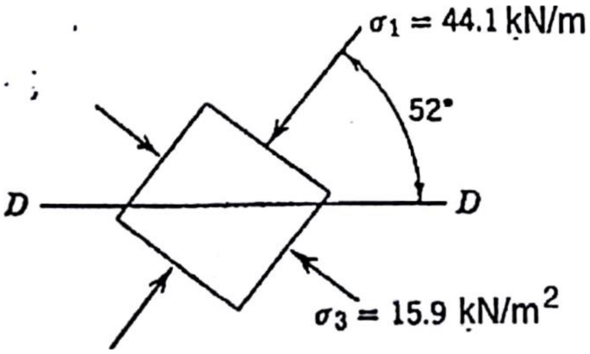
For a given in figure find the direction and magnitude of principal stresses

Case two: Given σ_θ and τ_θ, **required** σ_1 and σ_3

Solution

1. locate points (40,-10) and (20,10)
2. Erect diameter and draw Mohr circle, using these points to defined diameter, diameter location = (40+20)/ 2 = 30 kN/m^2, thus center location is (30, 0).
3. Draw line B' B' through point (40, -10) and parallel BB.
4. Read σ_1 and σ_3 from graph
5. Line though O_P and σ_1 give plane on which σ_1 acts.





Solution by equations

1. First make use of tact that sum of normal stresses is a constant:

$$\frac{\sigma_1 + \sigma_3}{2} = \frac{\Sigma\sigma_\theta}{2} = \frac{40 + 20}{2}$$

2. Use relation

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \sqrt{\left[\sigma_\theta - \left(\frac{\sigma_1 + \sigma_3}{2}\right)\right]^2 + [\tau_\theta]^2}$$

with either pair of given stresses

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \sqrt{[20 - 30]^2 + [10]^2} = \sqrt{200} = 14.14 \text{ kN/m}^2$$

3.

$$\sigma_1 = \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right) = 44.14 \text{ kN/m}^2$$

$$\sigma_3 = \left(\frac{\sigma_1 + \sigma_3}{2}\right) - \left(\frac{\sigma_1 - \sigma_3}{2}\right) = 15.86 \text{ kN/m}^2$$

4. Use stress pair in which σ_θ is largest; i.e. (40, −10)

$$\sin 2\theta = \frac{2\tau_\theta}{\sigma_1 - \sigma_3} = \frac{-20}{28.28} = -0.707$$

$$2\theta = -45^\circ$$

$$\theta = -22\tfrac{1}{2}^\circ$$

5. Angle from horizontal to major principal stress direction $= 30^\circ - \theta = 52\tfrac{1}{2}^\circ$.

4.5 Stress Increment Soil

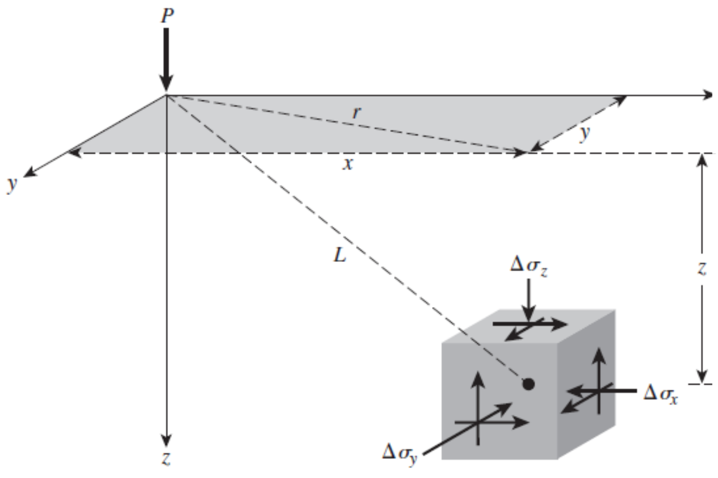
Construction of a foundation causes changes in the stress. The net stress increase in the soil depends on the load per unit area to which the foundation is subjected, the depth below the foundation at which the stress estimation is desired, and other factors. It is necessary to estimate the net increase of vertical stress in soil that occurs due to construction so that settlement can be calculated. The estimation of vertical stress is based on the theory of elasticity. The loads may include:

Point load	Uniformly loaded rectangular area
Line load	
Uniformly distributed vertical strip load	Uniformly loaded circular area
Linearly increasing vertical loading on a strip	Embankment type of loading

Although natural soil deposits, in most cases, are not fully elastic, isotropic, or homogeneous materials, calculations for estimating increases in vertical stress yield fairly good results for practical work.

4.5.1 Stresses Caused by a Point Load

Boussinesq (1883) solved the problem of stresses produced at any point in a homogeneous, elastic, and isotropic medium as the result of a point load applied on the surface of an infinitely large half-space.



if $r = \sqrt{x^2 + y^2}$ $\quad L = \sqrt{r^2 + z^2}$

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2 + z^2)^{5/2}} \quad = \Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2 + 1]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

Table .1 Variation of I_1 for Various Values of r/z for point load

r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.36	0.3521	1.80	0.0129
0.02	0.4770	0.38	0.3408	2.00	0.0085
0.04	0.4765	0.40	0.3294	2.20	0.0058
0.06	0.4723	0.45	0.3011	2.40	0.0040
0.08	0.4699	0.50	0.2733	2.60	0.0029
0.10	0.4657	0.55	0.2466	2.80	0.0021
0.12	0.4607	0.60	0.2214	3.00	0.0015
0.14	0.4548	0.65	0.1978	3.20	0.0011
0.16	0.4482	0.70	0.1762	3.40	0.00085
0.18	0.4409	0.75	0.1565	3.60	0.00066
0.20	0.4329	0.80	0.1386	3.80	0.00051
0.22	0.4242	0.85	0.1226	4.00	0.00040
0.24	0.4151	0.90	0.1083	4.20	0.00032
0.26	0.4050	0.95	0.0956	4.40	0.00026
0.28	0.3954	1.00	0.0844	4.60	0.00021
0.30	0.3849	1.20	0.0513	4.80	0.00017
0.32	0.3742	1.40	0.0317	5.00	0.00014
0.34	0.3632	1.60	0.0200		

Example (4.13)

Consider a point load P = 5 kN, calculate the vertical stress increase $\Delta\sigma_z$ at z = 0, 2 m, 4 m, 6 m, 10 m, and 20 m. Given x =3 m and y =4 m.

Solution

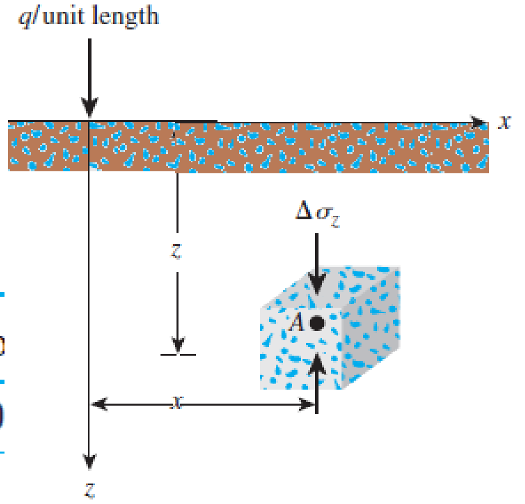
$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

the following table can be prepare

r (m)	z (m)	$\frac{r}{z}$	I_1	$\Delta\sigma_z = \left(\frac{P}{z^2}\right) I_1$ (kN/m²)
5	0	∞	0	**0**
	2	2.5	0.0034	**0.0043**
	4	1.25	0.0424	**0.0133**
	6	0.83	0.1295	**0.0180**
	10	0.5	0.2733	**0.0137**
	20	0.25	0.4103	**0.0051**

4.5.2 Vertical Stress Caused by a Vertical Line Load

The stresses increment due to line load can be calculated using the following equation:



$$\frac{\Delta\sigma_z}{(q/z)} = \frac{2}{\pi[(x/z)^2 + 1]^2}$$

Table .2 Variation of $\Delta\sigma_z/(q/z)$ with x/z for line lo

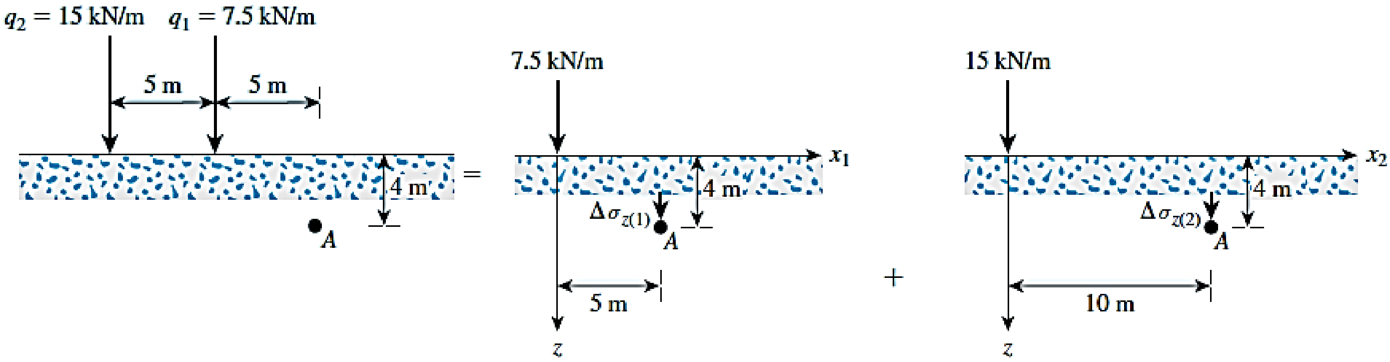
x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0	0.637	1.3	0.088
0.1	0.624	1.4	0.073
0.2	0.589	1.5	0.060
0.3	0.536	1.6	0.050
0.4	0.473	1.7	0.042
0.5	0.407	1.8	0.035
0.6	0.344	1.9	0.030
0.7	0.287	2.0	0.025
0.8	0.237	2.2	0.019
0.9	0.194	2.4	0.014
1.0	0.159	2.6	0.011
1.1	0.130	2.8	0.008
1.2	0.107	3.0	0.006

Example (4.14)

Figure shows two line loads on the ground surface. Determine the increase of stress at point A.

Solution

The total stress at A is



$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)}$$

$$\sigma_{z(1)} = \frac{2q_1z^3}{\pi(x_1^2 + z^2)^2} = \frac{(2)(7.5)(4)^3}{\pi(5^2 + 4^2)^2} = 0.182 \text{ kN/m}^2$$

$$\sigma_{z(2)} = \frac{2q_2z^3}{\pi(x_2^2 + z^2)^2} = \frac{(2)(15)(4)^3}{\pi(10^2 + 4^2)^2} = 0.045 \text{ kN/m}^2$$

$$\Delta\sigma_z = 0.182 + 0.045 = \mathbf{0.227 \text{ kN/m}^2}$$

4.5.3 Vertical and horizontal Stresses Caused by a Vertical Uniform Distributed Load on Circular Area

F: factor, can be find from the following figure

R: is the radius of the circular area

X: is the distance from the center of the circle to the point

Z: is the depth of the point.

Example (4.15)

For the soil with γ = 16.5 kN/m^2 and k_o = 0.5 loaded by Δq_s = 240 kN/m^2 over circular area of 6m indiameter. Find vertical and horizontal stresses at depth of 3m under the center of the loaded area.

Solution

Intial stresses: σ_v = 16.5 * 3 = 49.3 kN/m^2 , σ_h = 49.3 * 0.5 = 24.75 kN/m^2

Stress increment:

X = 0, R = 3, z = 3

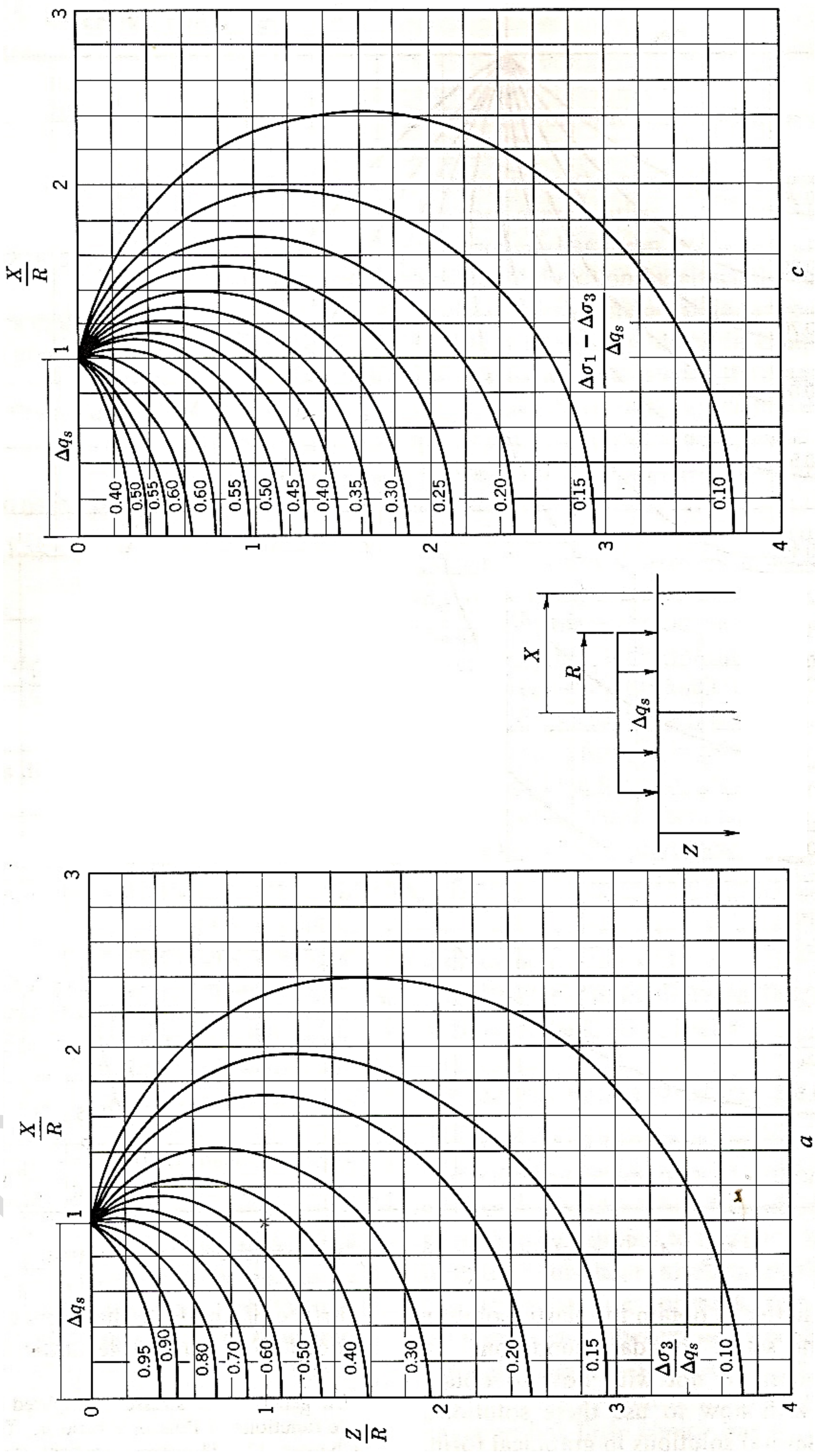
X/R = 0 , z/R = 1

Let $\Delta\sigma_v = \Delta\sigma_1$ and $\Delta\sigma_h = \Delta\sigma_3$

F = 0.64 for vertiacal and 0.64 -0.54 = 0.1

$\Delta\sigma_v$ = 0.64 * 240 = 153.6 kN/m^2, $\Delta\sigma_h$ = 0.1 * 240 = 24 kN/m^2

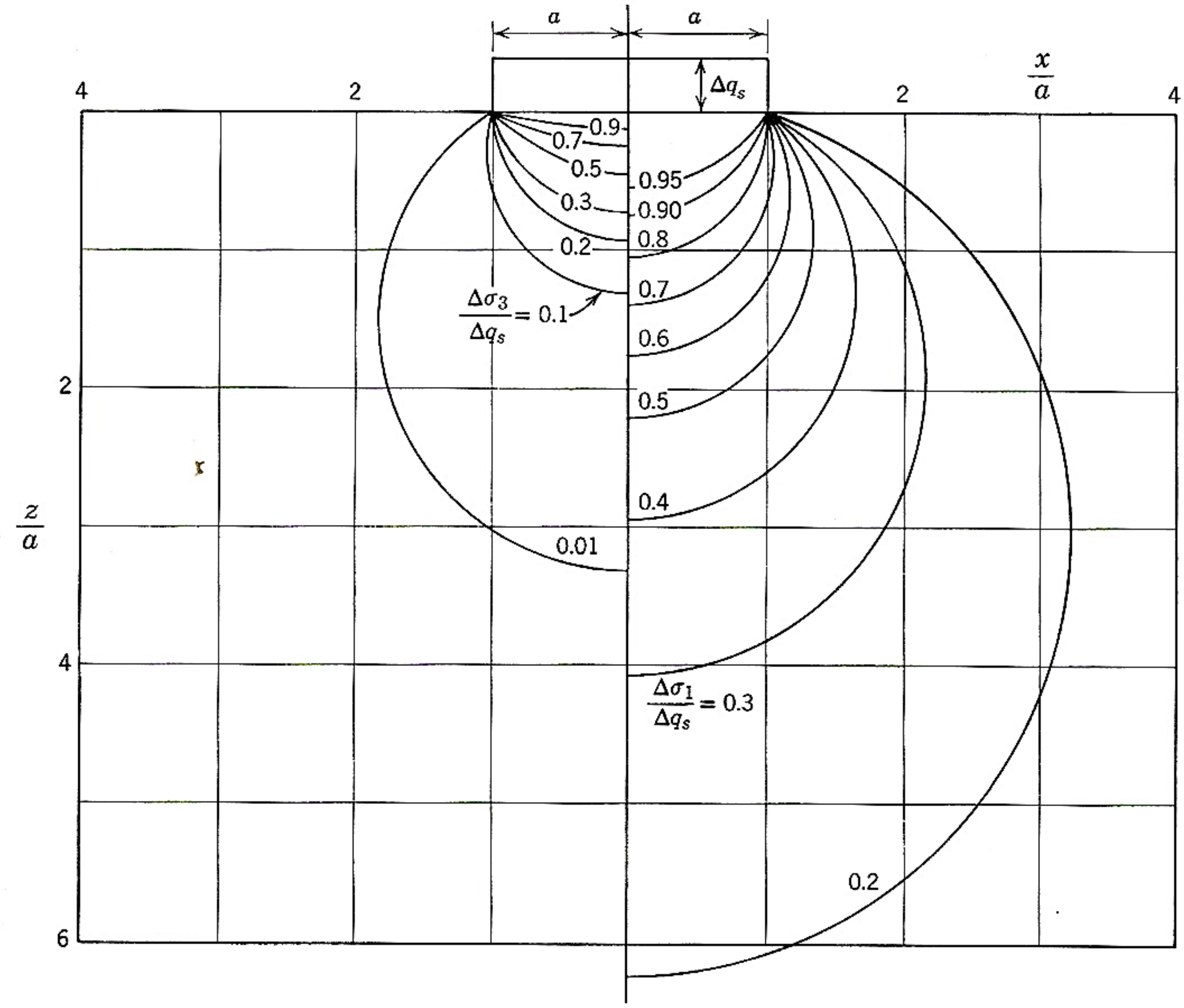
σ_{vf} = 153.6 + 49.3 = 202.9 kN/m^2, σ_{hf} = 24 + 24.75 = 48.75 kN/m^2



Stresses under uniform load on circular area.

4.5.4 Vertical and Horizontal Stressed Caused by a Vertical Uniform Distributed Load on Strip

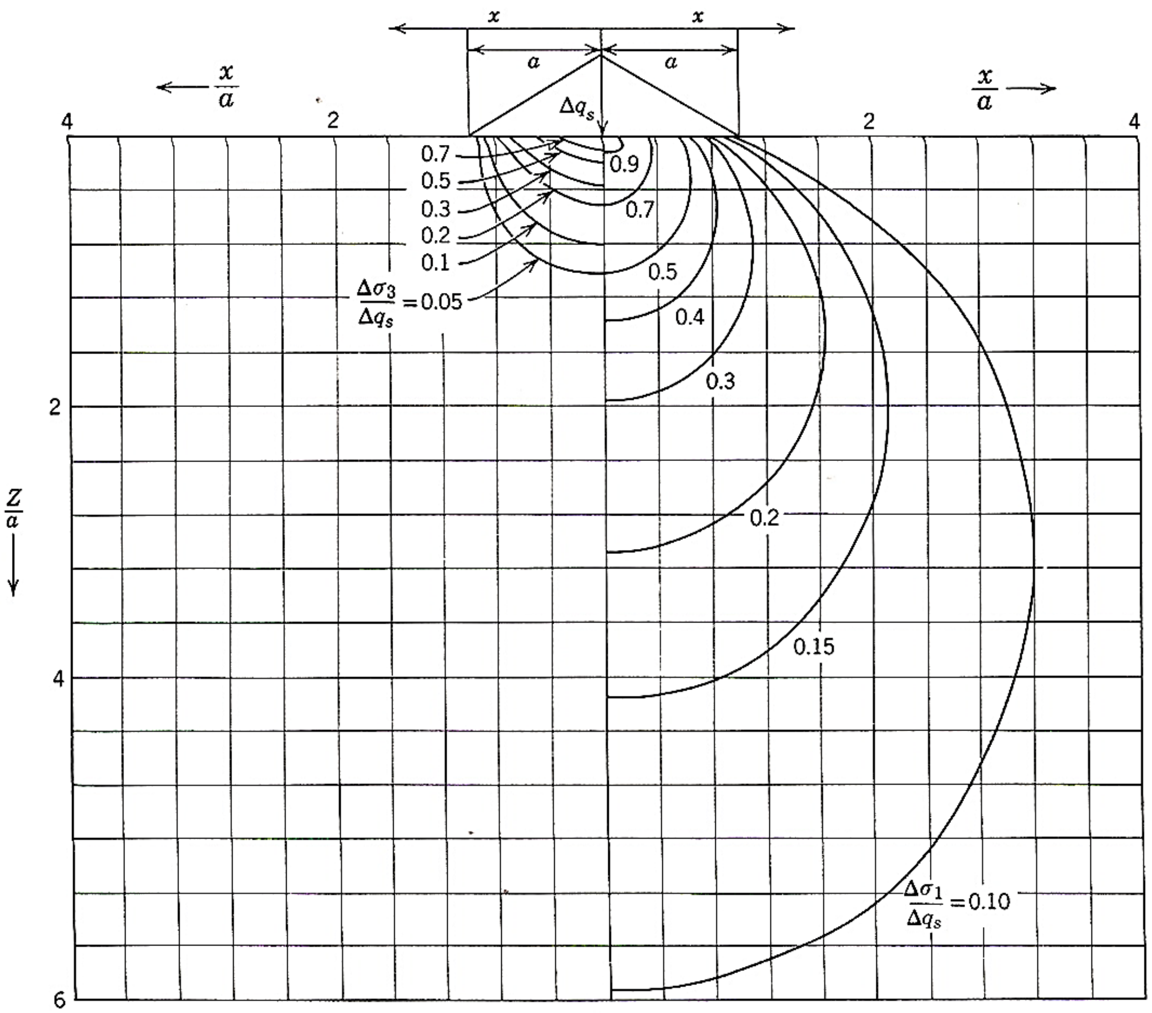
Use the following figure



Principal stresses under strip load.

4.5.5 Vertical and Horizontal Stressed Caused by a triangular Load on Strip

Use the following figure



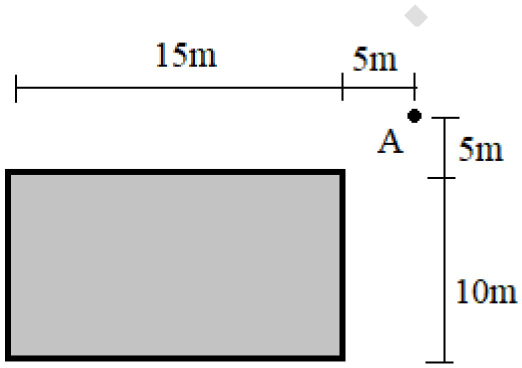
Principal stresses under triangular strip load.

4.5.6 Vertical Stress Caused by a Uniform distributed Load on Rectangular Surface

Use the following shape (Newmark chart) to find stress increment under the corner of rectangular area.

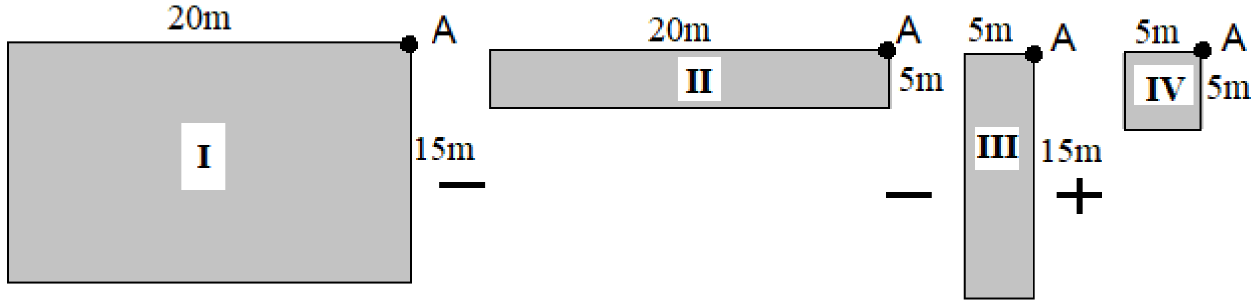
Example (4.16)

Find the vertical stress increment at depth of 10m below point A due to loading area of Δq_s = 240 kN/m^2.

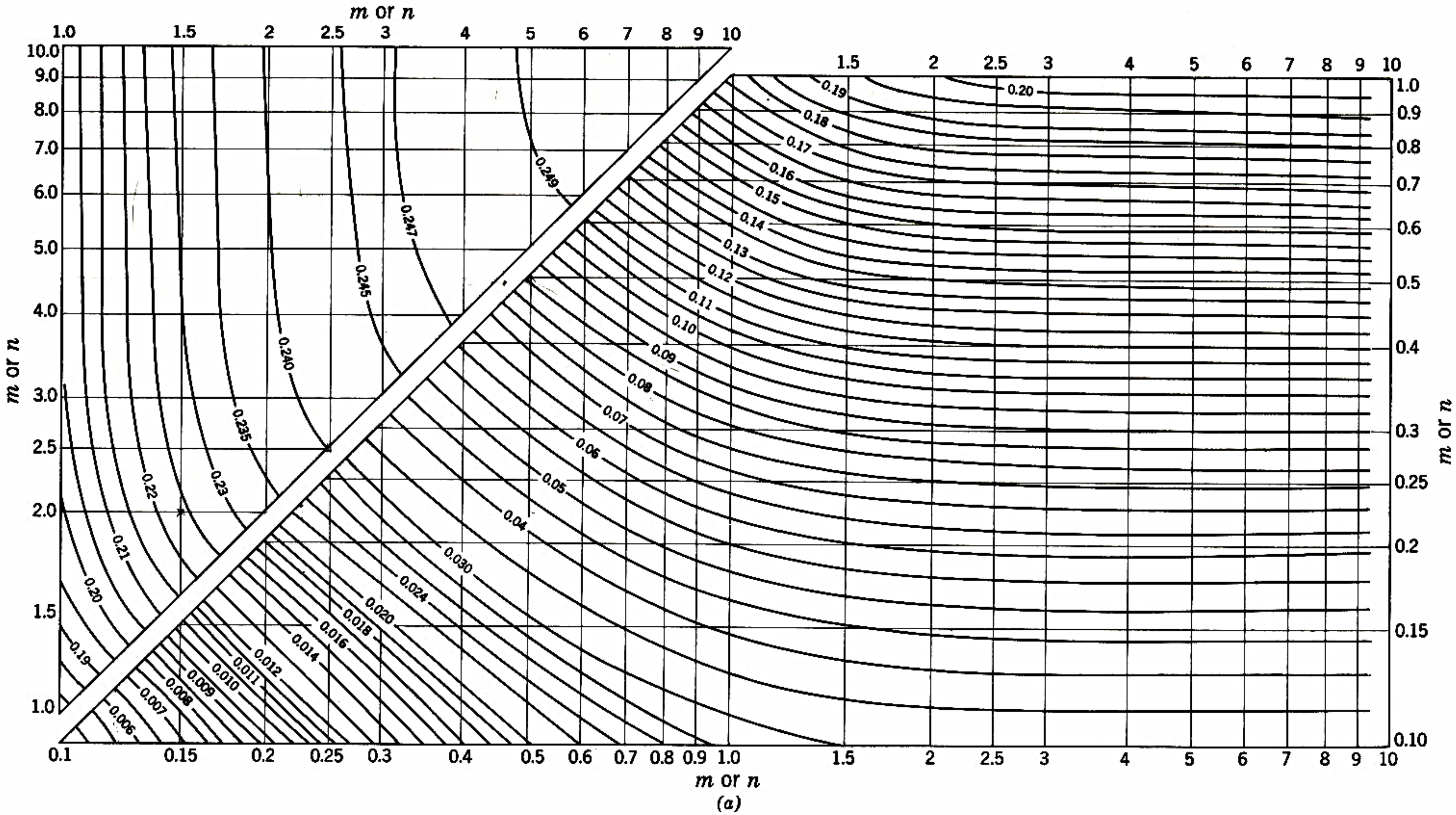


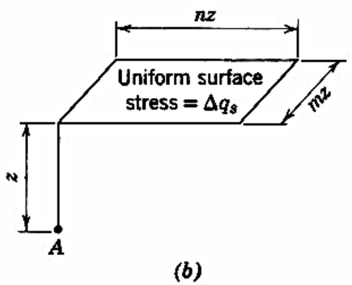
Solution

Devided the sahpe into:



Loading	m	n	Coeff	Δq_s	Factor	Stress increment, $\Delta\sigma$ kN/m^2
I	15/10=1.5	20/10=2.0	0.223	240	+1	+ 22.3
II	5/10=0.5	20/10=2.0	0.135	240	-1	- 13.5
III	15/10=1.5	5/10=0.5	0.131	240	-1	- 13.1
IV	5/10=0.5	5/10=0.5	0.085	240	+1	+ 8.5
Total stress increment						4.2

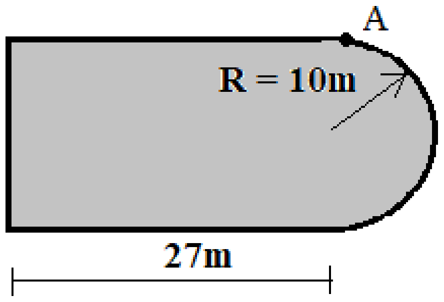


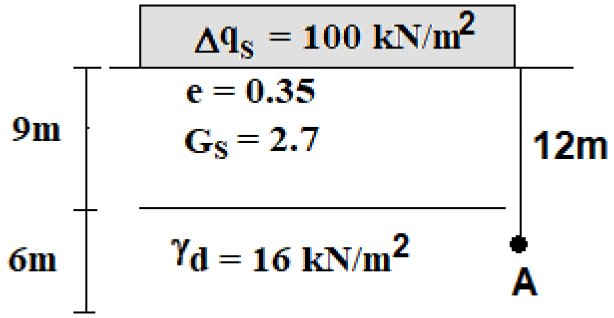


(a) Chart for use in determining vertical stresses below corners of loaded rectangular surface areas on elastic, isotropic material. Chart gives $f(m, n)$. (b) At point A, $\Delta\sigma_v = \Delta q_s \times f(m, n)$. (From Newmark, 1942)

Example (4.17)

For the loaded area a uniform pressure of 100 kN/m^2. Compute the vertical stress at depth of 12m below point A.



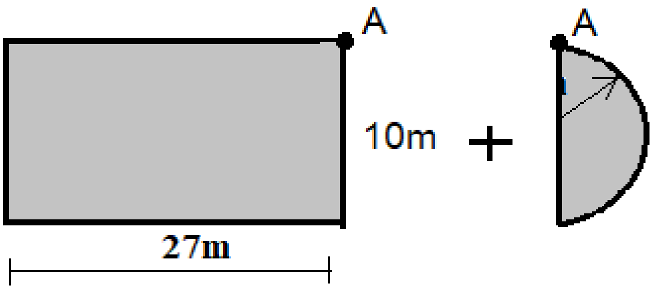


Solution

$$\gamma_d = \frac{G_s * \gamma_w}{1+e} = \frac{2.7 * 9.81}{1+0.35} = 19.62\ kN/m^2$$

σ_v @ 12m = 9 * 19.62 + 16 * 3 = 224.58 kN/m^2

Devided the area into



For circular area

X/R = 1 , z/R = 1.2

F = 0.3 for vertiacal stress = $\Delta\sigma_v$ = 0.3 * 0.5 * 100 = 15 kN/m^2

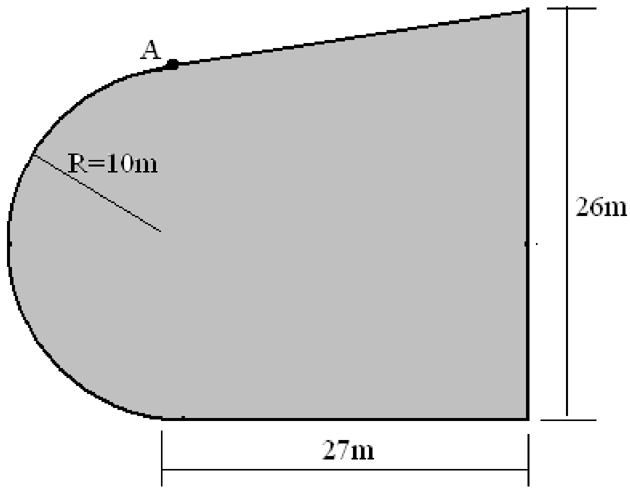
For rectangular area

n = 1.66 m = 2.25, coeff = 0.23 ∴ $\Delta\sigma_v$ = 0.23 * 100 = 23 kN/m^2,

σ_{vf} = 15 + 23 + 224.58 = 262.58 kN/m^2

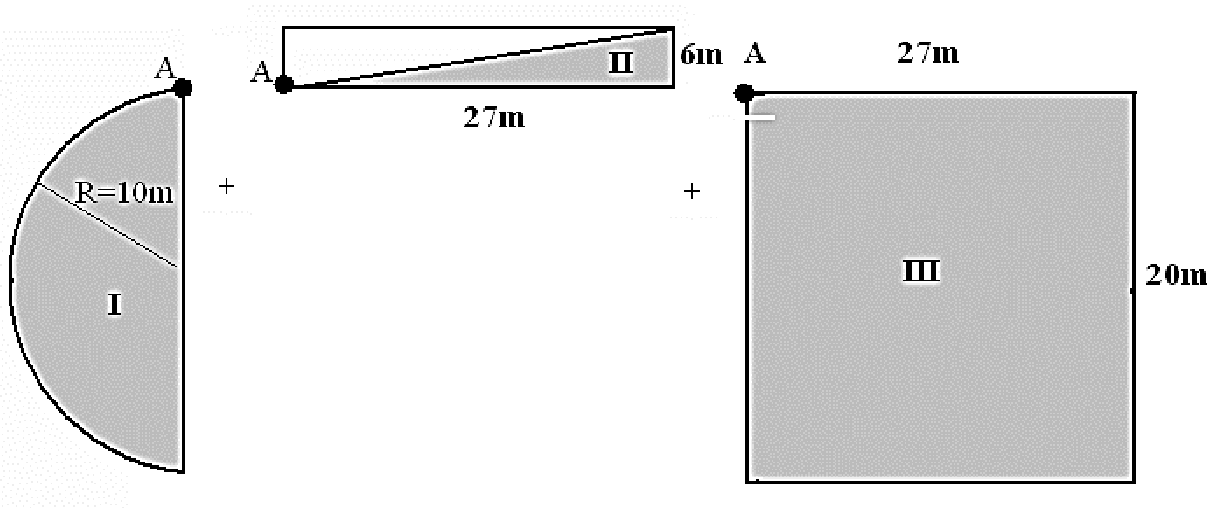
Example (4.18)

For the loaded area with uniform pressure on the ground surface with Δqs=100 kN/m2 as shown in figures. Compute the increment in vertical stresses at 5m below point A.



Solution

Devided the area into

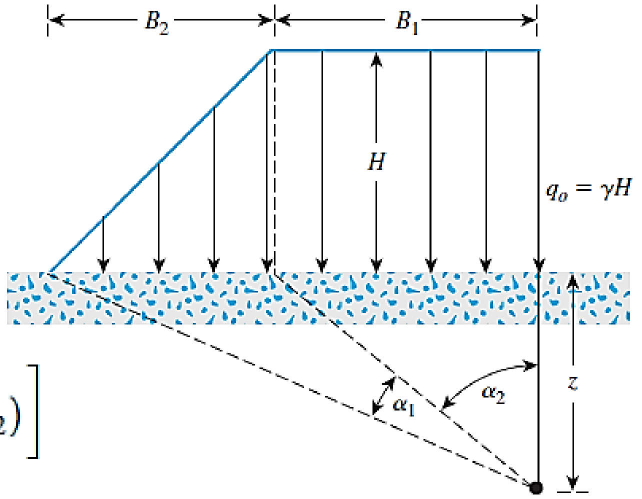


Loading	X/R	Z/R	Coeff.	Δq_s	Factor	Stress increment, $\Delta\sigma$ kN/m^2
I	1/10=0.1	5/10=0.5	0.9	100	0.5	45

Loading	m	n	Coeff.	Δqs	Factor	Stress increment, Δσ kN/m2
II	27/5=5.4	6/5=1.2	0.215	100	0.5	10.75
III	27/5=5.4	20/5=4	0.248	100	1	24.8
Total stress increment						80.55

4.6 Vertical Stress Due to Embankment Loading

The figure shows the cross section of an embankment of height H. For this two dimensional loading condition the vertical stress increase may be expressed as



$$\Delta\sigma_z = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right]$$

Where, $q_o = \gamma H$

γ = unit weight of the embankment soil

H = height of the embankment

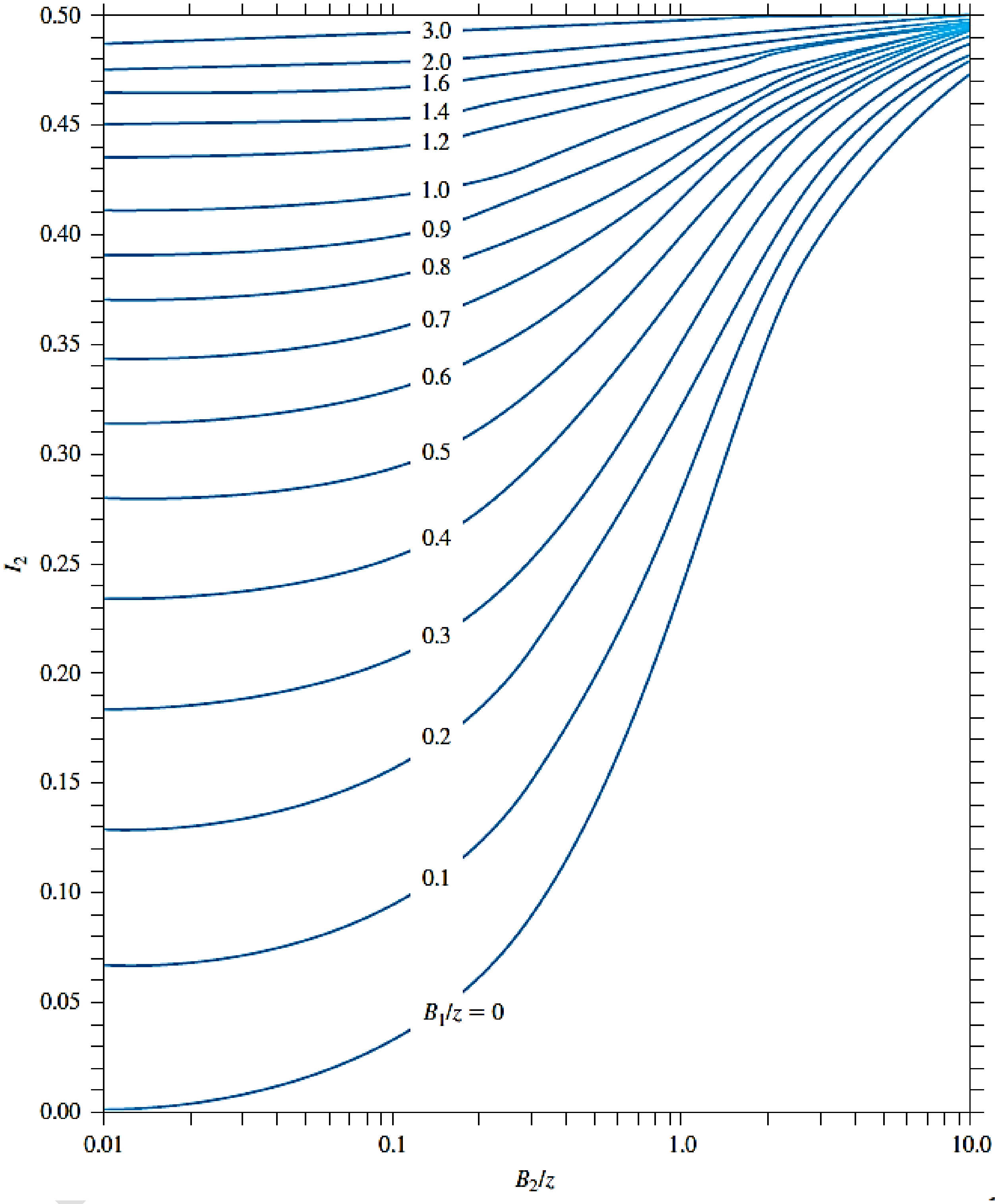
$$\alpha_1\,(\text{radians}) = \tan^{-1}\left(\frac{B_1 + B_2}{z}\right) - \tan^{-1}\left(\frac{B_1}{z}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right)$$

$\Delta\sigma_z = q_o\, I_2$

Where, I_2 = a function of B_1/z and B_2/z.

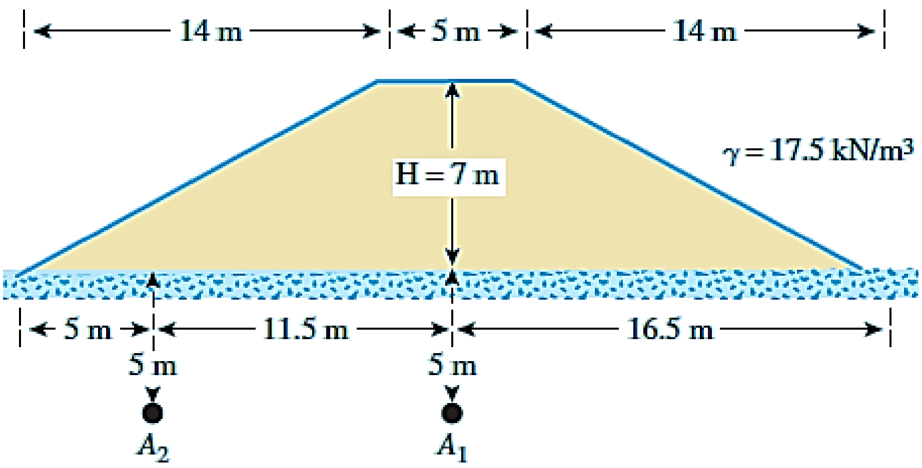
The variation of I_2 with B_1/z and B_2/z is shown in Figure



Osterberg's chart for determination of vertical stress due to embankment loading

Example (4.19)

An embankment is shown in Figure. Determine the stress increase under the embankment at points A1 and A2.



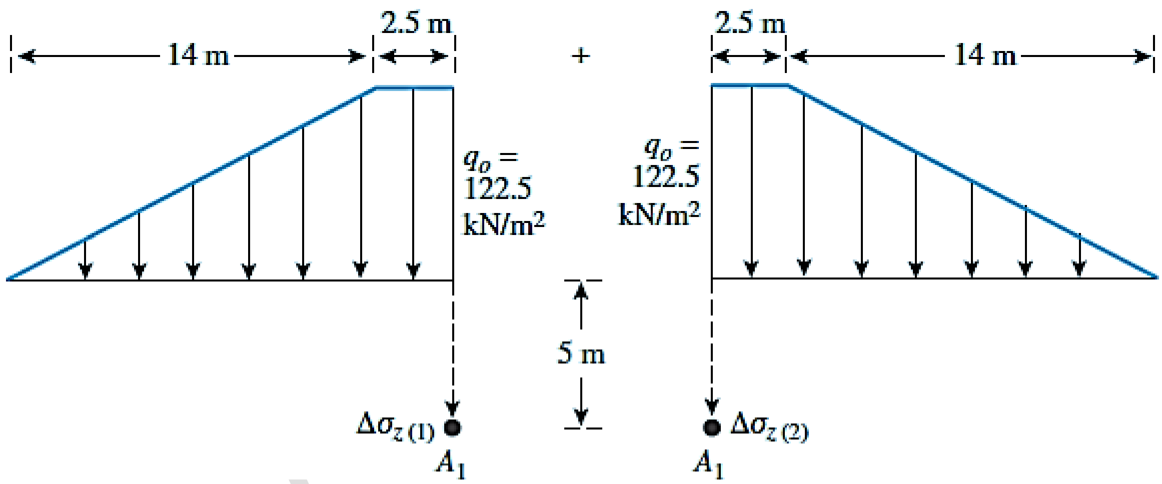
Solution

Stress Increase at A_1

$\gamma H = 17 * 5 = 122.5 \text{ kN/m}^2$

The left side of the figure below indicates that $B_1 = 2.5$ m and $B_2 = 14$ m. So,

$$\frac{B_1}{z} = \frac{2.5}{5} = 0.5; \frac{B_2}{z} = \frac{14}{5} = 2.8$$

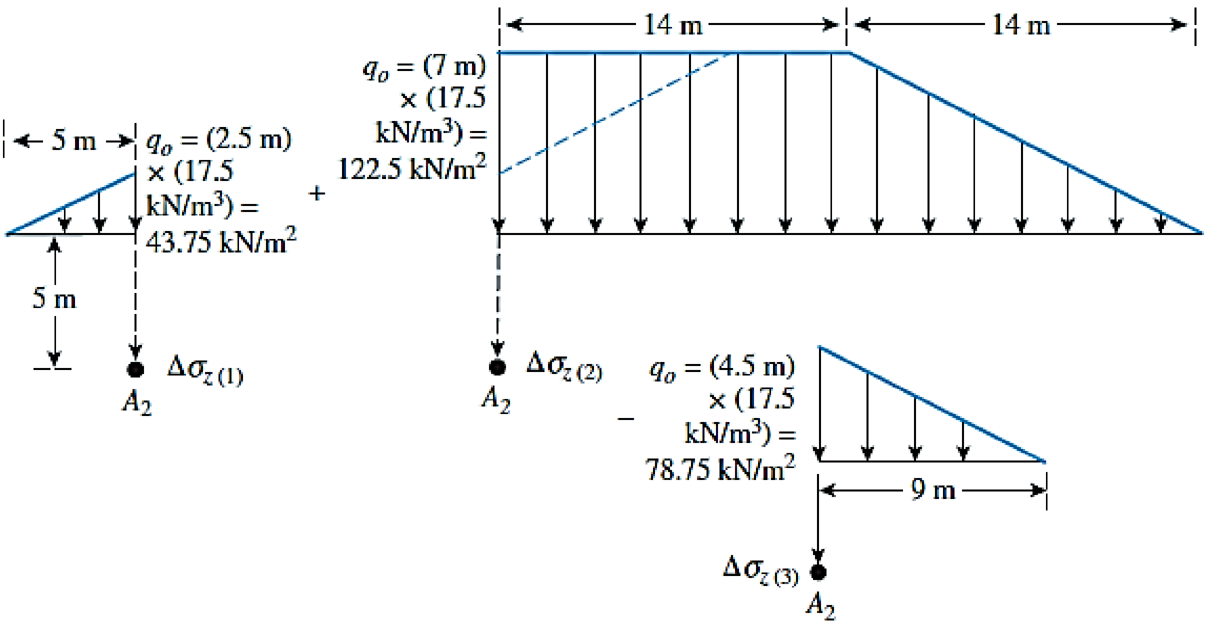


According to Osterberg's chart, in this case, $I_2 = 0.445$ Because the two sides in this figure are symmetrical, the value of I_2 for the right side will also be 0.445.

So
$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = q_o[I_{2(\text{Left})} + I_{2(\text{Right})}]$$
$$= 122.5[0.445 + 0.445] = \mathbf{109.03 \text{ kN/m}^2}$$

Stress Increase at A_2

Refer to Figure below. For the left side, $B_2 = 5$ m and $B_1 = 0$.



So, $\frac{B_2}{z} = \frac{5}{5} = 1; \frac{B_1}{z} = \frac{0}{5} = 0$

According to Osterberg's chart, for these values of B_2/z and B_1/z, $I_2 = 0.24$. So,

$$\Delta\sigma_{z(1)} = 43.75(0.24) = 10.5 \text{ kN/m}^2$$

For the middle section,

$$\frac{B_2}{z} = \frac{14}{5} = 2.8; \frac{B_1}{z} = \frac{14}{5} = 2.8$$

Thus, $I_2 = 0.495$. So, $\Delta\sigma_{z(2)} = 0.495(122.5) = 60.64 \text{ kN/m}^2$

For the right side,

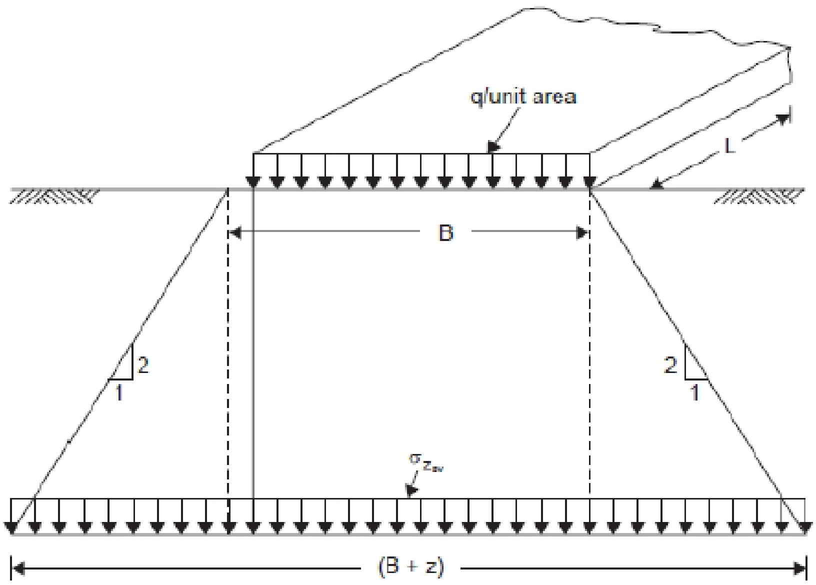
$$\frac{B_2}{z} = \frac{9}{5} = 1.8; \frac{B_1}{z} = \frac{0}{5} = 0$$

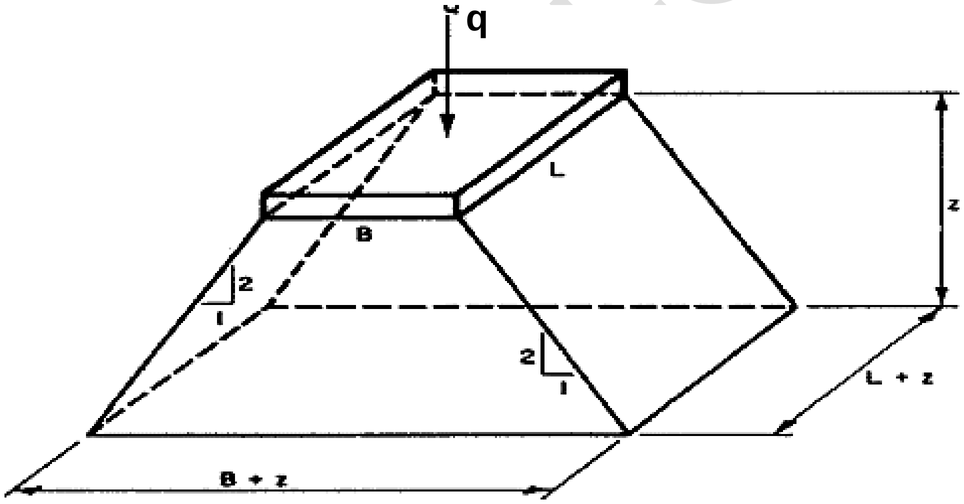
and $I_2 = 0.335$ So, $\Delta\sigma_{z(3)} = (78.75)(0.335) = 26.38 \text{ kN/m}^2$

Total stress increase at point A_2 is

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} - \Delta\sigma_{z(3)} = 10.5 + 60.64 - 26.38 = \mathbf{44.76\ kN/m^2}$$

4.7 Method 2:1





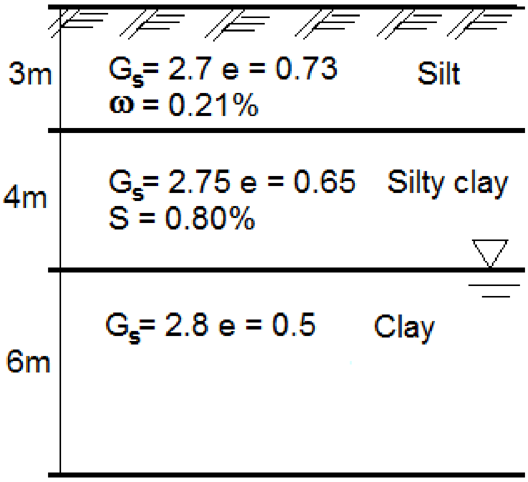
$$\Delta\sigma_z = \frac{Q}{(B+Z)(L+Z)}$$ Rectangular area

$$\Delta\sigma_z = \frac{Q}{(B+Z)^2}$$ Square area

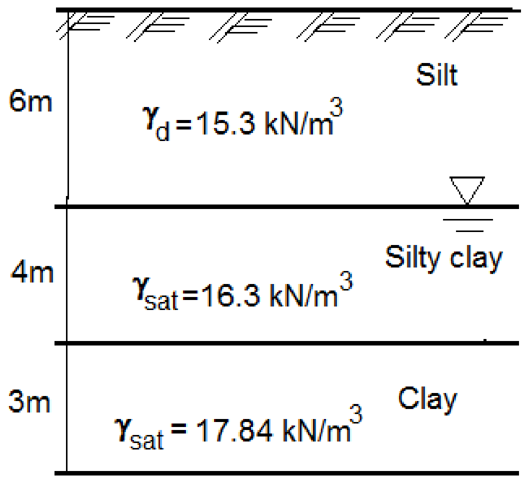
$$\Delta\sigma_z = \frac{Q}{\frac{\pi}{4}(D+Z)^2}$$ Circular area

Homework Chapter 4

4.1 For the soil profile shown calculate the total, effective vertical stresses and pore water pressure.

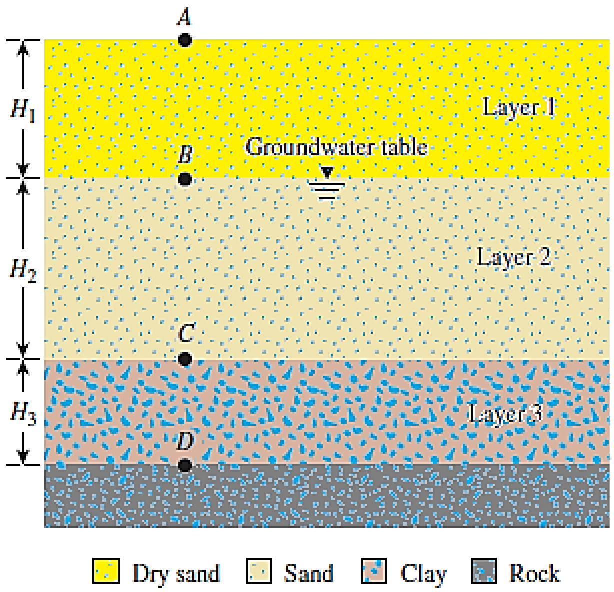


4.2 Draw the stresses σ, u, and σ' with depth for the profile shown.



4.3 A soil profile consisting of three layers is shown in the figure. Calculate the values of σ, u, and σ' at points A, B, C, and D and plot the variations of σ, u, and σ' with depth. Characteristics of layers 1, 2, and three are given below:

Layer no.	Thickness	Soil parameters
1	$H_1 = 2.1$ m	$\gamma_d = 17.23$ kN/m^3
2	$H_2 = 3.66$ m	$\gamma_{sat} = 18.96$ kN/m^3
3	$H_3 = 1.83$ m	$\gamma_{sat} = 18.5$ kN/m^3



4.4 Redo problem 4.3 for the same profile but with following soil characteristics.

Layer no.	Thickness	Soil parameters
1	$H_1 = 5$ m	$e = 0.7$; $G_s = 2.69$
2	$H_2 = 8$ m	$e = 0.55$; $G_s = 2.7$
3	$H_3 = 3$ m	$w = 38\%$; $e = 1.2$

4.5 Redo problem 4.3 for the same profile but with following soil characteristics.

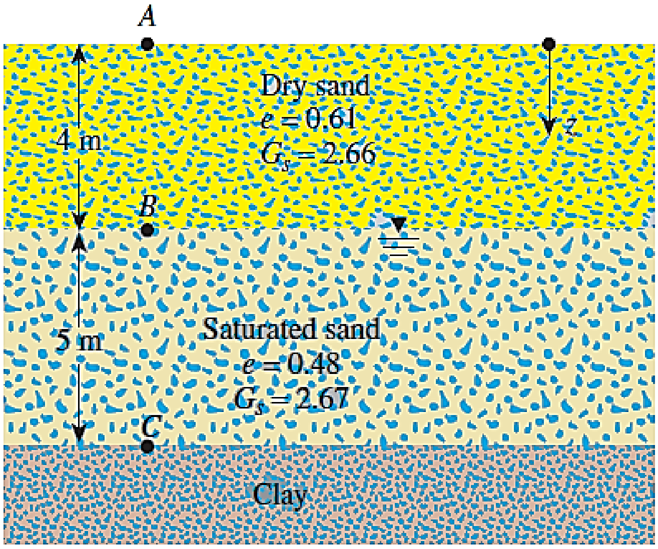
Layer no.	Thickness	Soil parameters
1	$H_1 = 3$ m	$\gamma_d = 16$ kN/m^3
2	$H_2 = 6$ m	$\gamma_{sat} = 18$ kN/m^3
3	$H_3 = 2.5$ m	$\gamma_{sat} = 17$ kN/m^3

4.6 Consider the soil profile in Problem 4.2. What is the change in effective stress at point C if:

a. The water table drops by 2 m
b. The water table rises to the surface up to point A
c. Water level rises 3 m above point A due to flooding

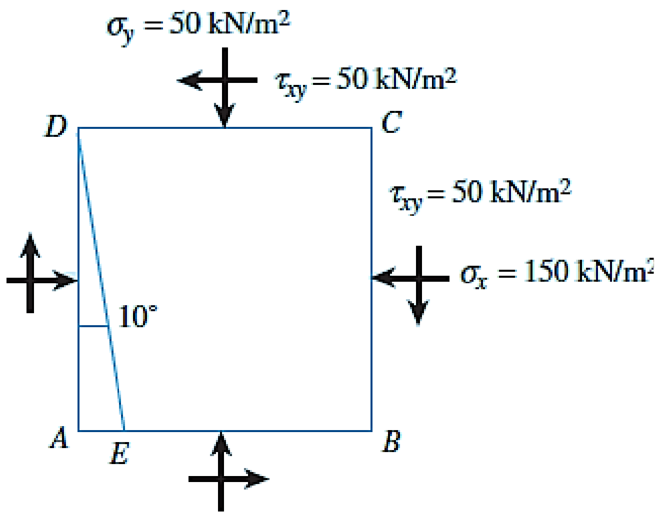
4.7 Consider the soil profile shown in Figure:

a. Calculate the variations of σ, u, and σ' at points A, B, and C.
b. How high should the groundwater table rise so that the effective stress at C is 111 kN/m^2



4.8 For the stressed soil element shown in Figure, determine

a) Major principal stress
b) Minor principal stress
c) Normal and shear stresses on the plane DE

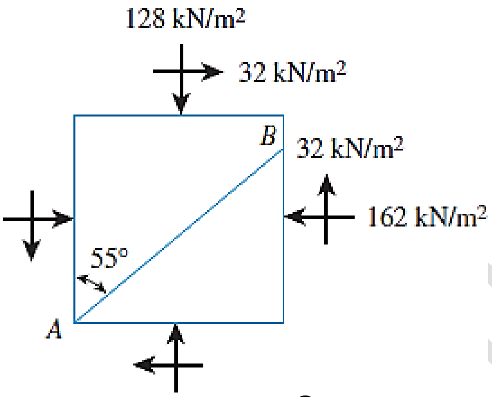


(Ans.) a) Major principal stress = 170.7 kN/m^2, b) Minor principal stress = 29.3 kN/m^2, c) Normal stress = 164 kN/m^2, Shear stress = - 29.9 kN/m^2

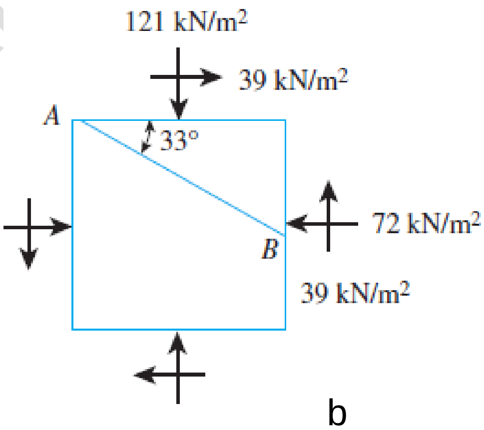
4.9 A soil elements are shown in Figures.

Determine the following (by drawing and equations):

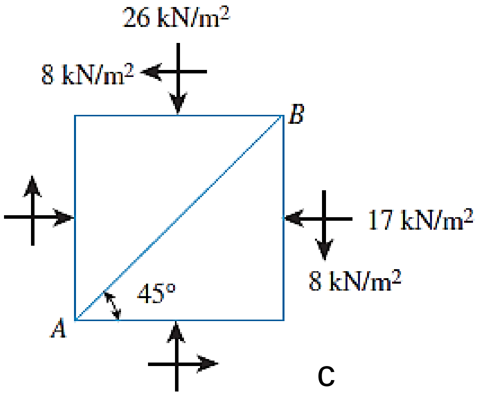
1. Maximum and minimum principal stresses
2. Normal and shear stresses on plane AB



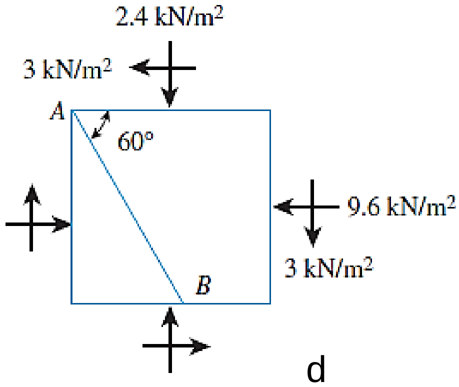
a



b



c



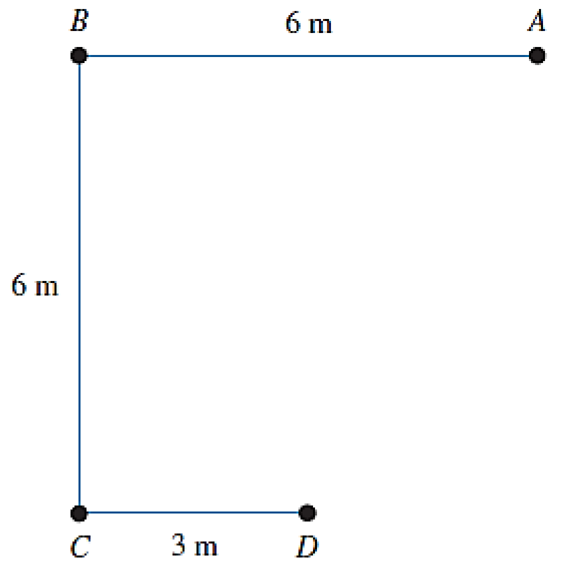
d

(Ans.) a. 1. $\sigma_1 = 181.23$ kN/m^2; $\sigma_3 = 108.76$ kN/m^2
2. $\sigma_n = 169.25$ kN/m^2; $\tau_n = -26.92$ kN/m^2

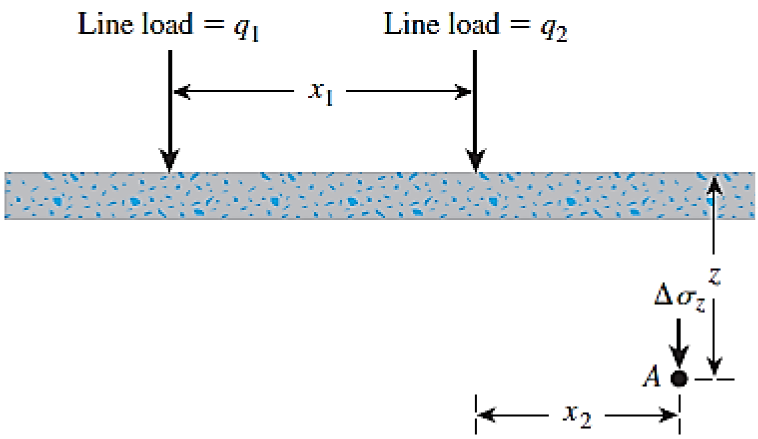
c. 1. $\sigma_1 = 30.68$ kN/m^2; $\sigma_3 = 12.32$ kN/m^2
2. $\sigma_n = 13.53$ kN/m^2; $\tau_n = 4.55$ kN/m^2

4.10 Point loads of magnitude 100, 200, and 400 kN act at B, C, and D, respectively. Determine the increase in vertical stress at a depth of 6 m below the point A. Use Boussinesq's equation.

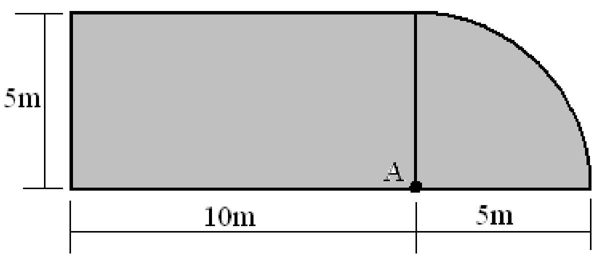
Ans. 1,127 kN/m^2



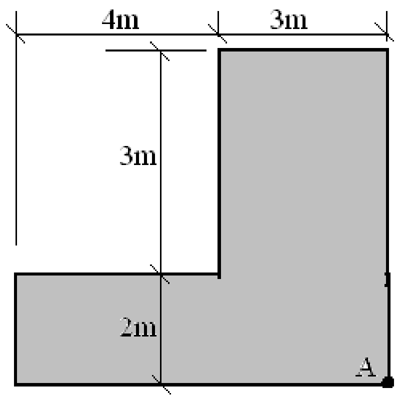
4.11 Determine the vertical stress increase, $\Delta\sigma_z$, at point A with the following values: q_1 = 90 kN/m; q_2 = 325 kN/m; x_1 = 4 m; x_2 = 2.5 m; z = 3 m.



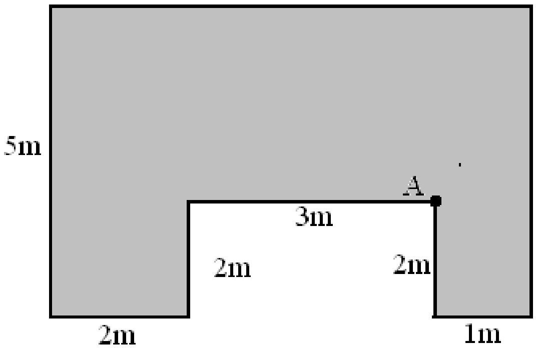
4.12 For the loaded area with uniform pressure on the ground surface with Δqs=100 kN/m2 as shown in figures. Compute the increment in vertical stresses at 5m below point A.



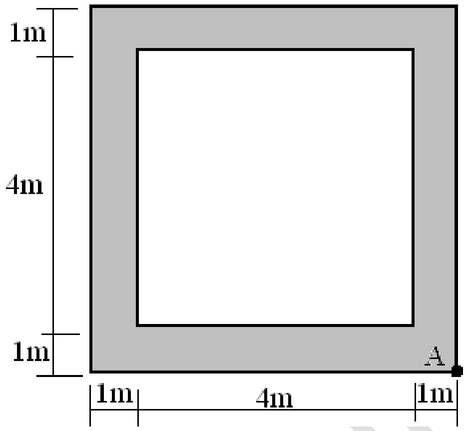
A



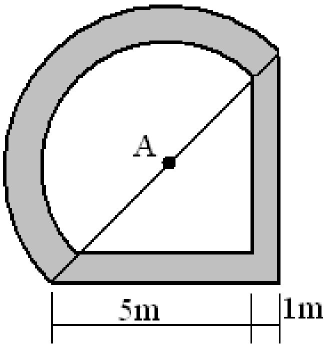
B



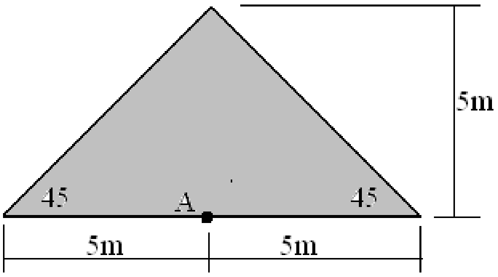
C



D



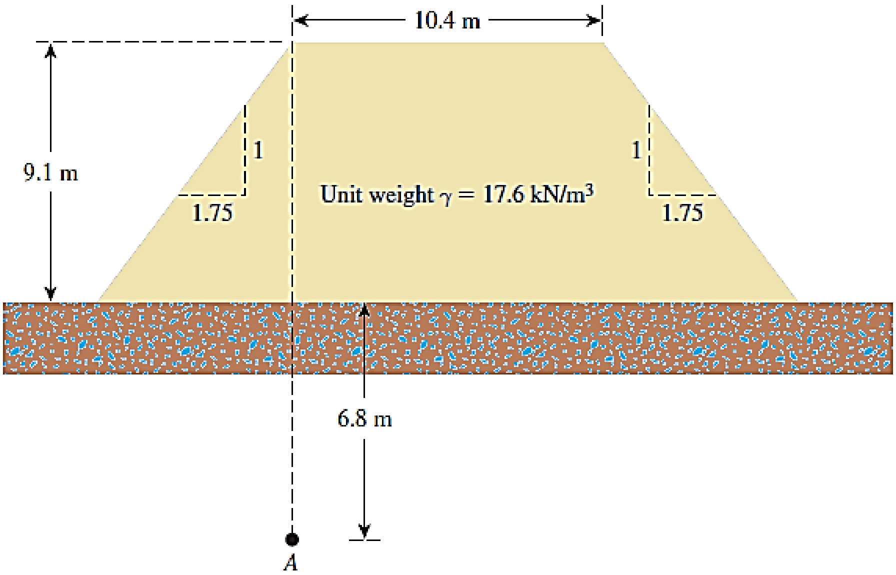
E



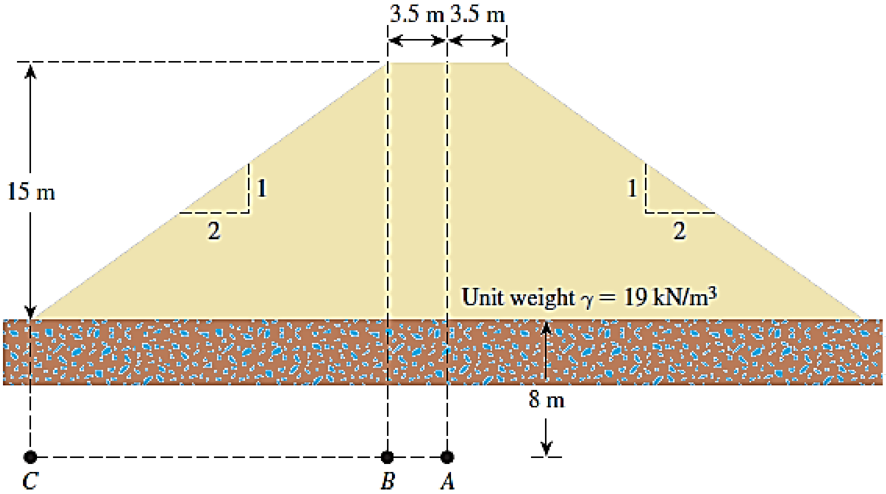
F

Ans. A = 36 kN/m^2, B = 16.5 kN/m^2, C = 80.55 kN/m^2 D = 1.8 kN/m^2 E = 26 kN/m^2 F = 10.6 kN/m^2

4.13 An earth embankment is shown in Figure. Determine the stress increase at point A due to the embankment load.



4.14 For the embankment loading shown in Figure, determine the vertical stress increases at points A, B, and C.



4.15 A circular area on the ground surface is subjected to a uniformly distributed load, q = 105 kN/m^2. If the circular area has a radius, R = 3.6 m, determine the vertical stress increase, at points 0, 1.2, 2.4, 4.8 and 9.6 m below the ground surface along the centerline of the circular area.

4.16 A circular area of radius, R = 5 m subjected to a uniformly distributed load, q = 380 kN/m^2. Determine the vertical stress increases 3 m below the loaded area at radial distances, r = 0, 1, 3, 5, and 7 m.

4.17 Refer to Figure. A rectangular area is uniformly loaded by q = 225 kN/m^2. Using Newmark's chart, determine the increase in vertical stress, at points A, B and C at depth of 3m.